Mathematical and Numerical Modeling of Tsunamis in Nearshore Environment: Present and Future

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Flooding and Erosion: Southern Banda Aceh (Gleebruk: 31miles southwest of Banda Aceh)

4/12/2004

1/2/2005

Infrastructure and Building Damage

1960, Chilean tsunami (Mw = 8.6): Isla Chiloe, Chile about 200 mortalities (left); Hilo, Hawaii 61 mortalities (right)

2004 Sumatra tsunamis: Kalmunai, Sri Lanka

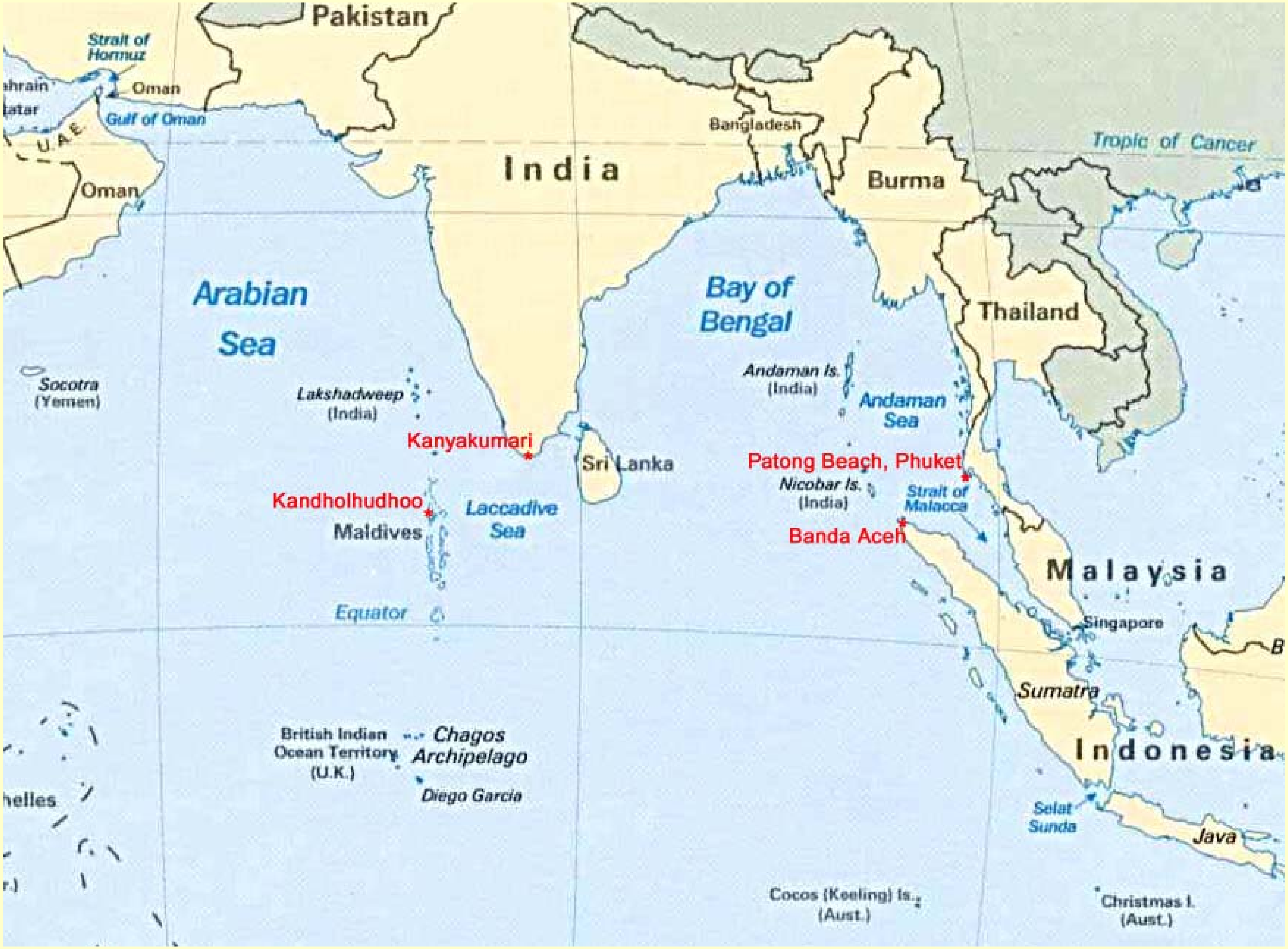
1946 Aleutian tsunami in Hilo, Hawaii. 96 people died, $26 million damage.

Debris Flows and structure damage

2004 Sumatra tsunamis: **Banda Aceh, Indonesia**

1983 Sea of Japan Tsunami

December 26, 2004 Indian Ocean Tsunamis



General Characterization of Tsunamis in the Nearshore and Onshore Region

- Breaking waves containing different time and length scales
- Turbulent flows
- Sediment (and debris) laden flows
- 3D flows strongly affected by bathymetry, topography, and surface conditions

Present state of modeling tsunami in nearshore and onshore environment

Non-linear Shallow Water Equations in Cartesian Coordinates:

$$\frac{\partial \zeta}{\partial t}+\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}=0$$

$$\frac{\partial P}{\partial t}+\frac{\partial}{\partial x}\left(\frac{P^2}{H}\right)+\frac{\partial}{\partial y}\left(\frac{PQ}{H}\right)+gH\frac{\partial \zeta}{\partial x}+\tau_x H=0$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{PQ}{H}\right)+\frac{\partial}{\partial y}\left(\frac{Q^2}{H}\right)+gH\frac{\partial \zeta}{\partial y}+\tau_y H=0$$

Bottom Frictional stress:

$$\tau_x=\frac{gn^2}{H^{10/3}}P(P^2+Q^2)^{1/2}$$

$$\tau_y=\frac{gn^2}{H^{10/3}}Q(P^2+Q^2)^{1/2}$$

$n =$ Manning's coefficient

ζ = free surface elevation

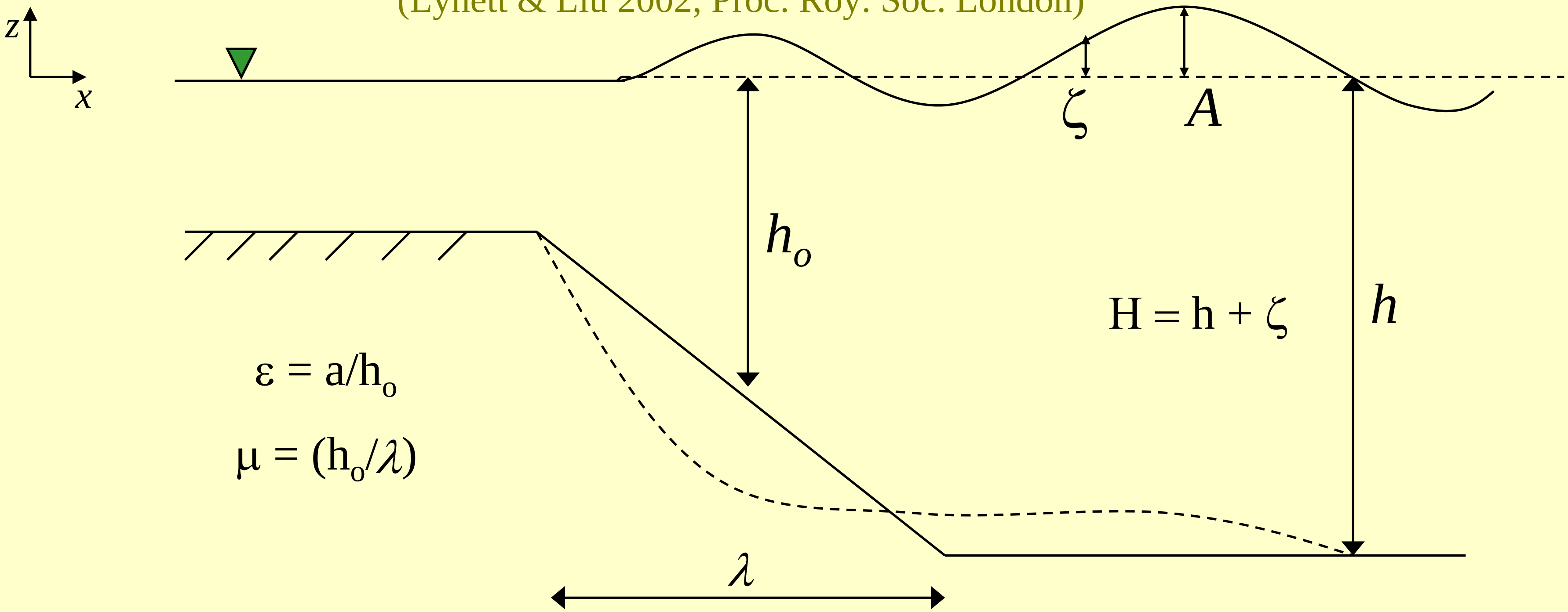
P = depth integrated volume flux in the x- direction

Q = depth integrated volume flux in the y- direction

H = total water depth

Other higher-order depth-integrated wave equations

(Lynett & Liu 2002, Proc. Roy. Soc. London)



Continuity Equation (Dimensionless)

$$\frac{1}{\varepsilon} H_t + \nabla \cdot (H u_\alpha) + O(\mu^2, \varepsilon\mu^2, \varepsilon^2\mu^2, \varepsilon^3\mu^2) = O(\mu^4)$$

Momentum Equation (Dimensionless)

$$u_{\alpha t} + \varepsilon u_\alpha \cdot \nabla u_\alpha + \nabla \zeta + O(\mu^2, \varepsilon\mu^2, \varepsilon^2\mu^2, \varepsilon^3\mu^2) = O(\mu^4)$$

[Complete](#) Equations

Some Important Length- and Time-scales for Submarine Earthquake Generated Waves

Fault area (Width x Length)

- 1960 Chilean tsunami: 200km x 800km
- 1964 Alaskan tsunami: 100km x 700km
- 2003 Algerian tsunami: 20km x 36km
- 2004 Sumatra tsunami: 200km x 1200km

Maximum fault displacement (dislocation)

- 1960 Chilean tsunami: 24 m
- 2003 Algerian tsunami: 1m
- 2004 Sumatra tsunami: 6 m

Resulting in an initial surface profile mimicking the seafloor deformation with a typical wavelength λ ~ O(10 – 100 km) and an amplitude A ~ O(1 – 10 m). For a typical water depth h ~ 1 – 4 km,

$$\varepsilon = A/h \sim O(10^{-2} - 2.5\times 10^{-4}), \qquad \mu^2 = \left(h/\lambda\right)^2 \sim O(10^{-1} - 10^{-4})$$

$$U_r = \varepsilon/\mu^2 \sim O(10^2 - 2.5\times 10^{-3})$$

The linear, non-dispersive wave theory seems is suitable to describe the initial propagation of seismically generated tsunami.

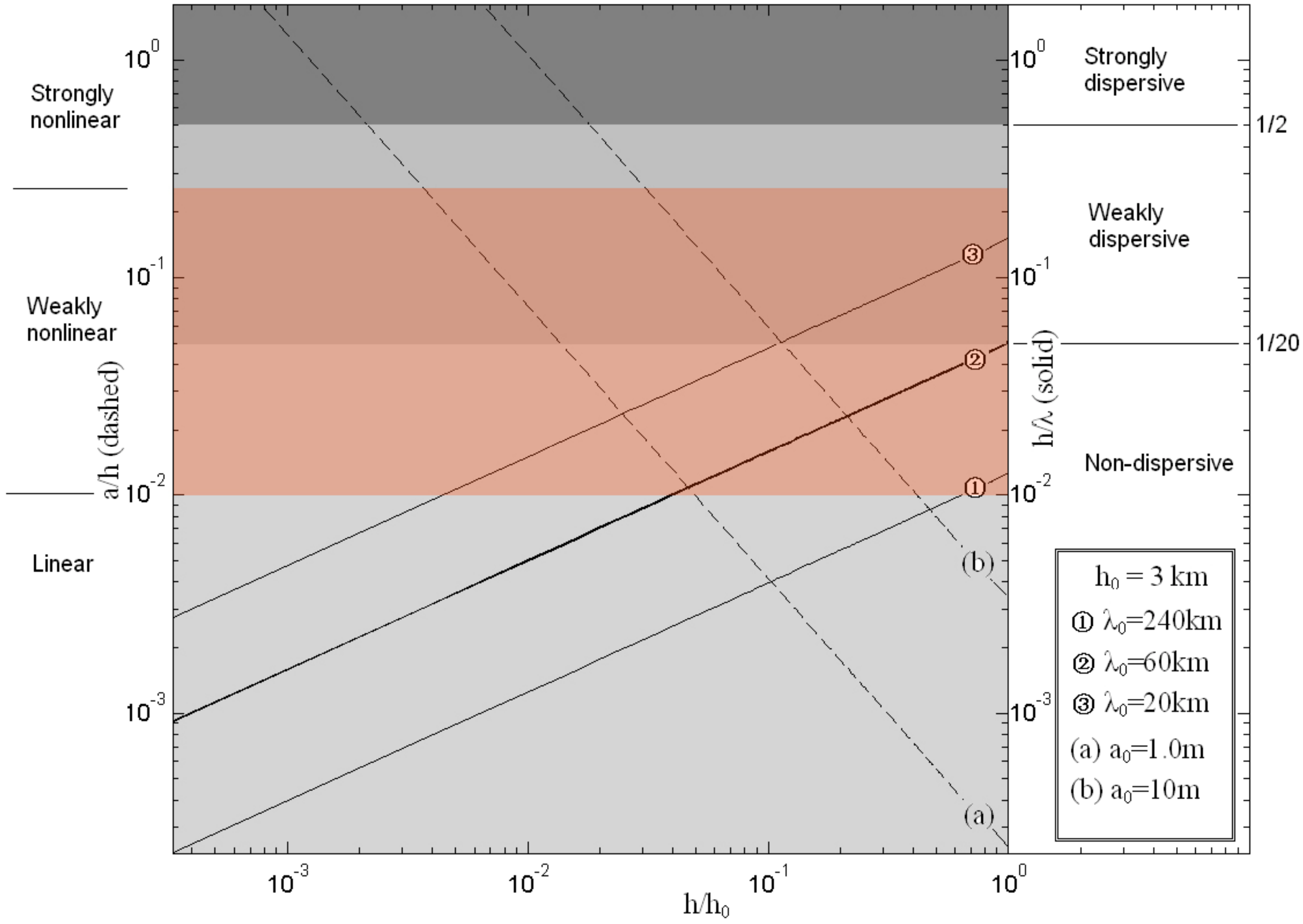
In the coastal region the wave amplitude and the wavelength must be rescaled **according to water depth:**

$$A \propto h^{-1/4},\ \lambda \propto h^{1/2},\ \varepsilon = A/h \propto h^{-5/4},\ \mu = h/\lambda \propto h^{1/2}$$

$$U_r = \varepsilon/\mu^2 \propto h^{-9/4}$$

This suggest that as the tsunami propagates into the coastal region, the importance of the nonlinearity will increase and that of the frequency dispersion will decrease with the decreasing water depth. Hence, in certain coastal region the Boussinesq wave theory might be necessary. However, in a very shallow depth, the nonlinearity dominates and the nonlinear shallow water wave theory becomes more adequate.

Choice of models based on the dispersion relationship



What is the limitation of the linear and non-dispersive wave theory?

By examining the validity of the perturbation solution for the 1D *KdV* equation, we know that

$$\text{if } U_r \;\square\; O(1),\ (g/h)^{1/2} t \;\square\; (\varepsilon\mu)^{-1};$$

$$\text{if } U_r \;\square\; O(1),\ (g/h)^{1/2} t \;\square\; \mu^{-3}.$$

The maximum distance is $x/h \sim (g/h)^{1/2} t$

Choice of Approximate Theories for Modeling Tsunami Propagation

U_r	$\tau = t\sqrt{g/h}$	Approximate Equation
$\square\ 1$	$\square\ (\varepsilon\mu)^{-1}$	Linear nondispersive
	$\sim O(\varepsilon\mu)^{-1}$	Non-linear nondispersive
$\square\ 1$	$\square\ \mu^{-3}$	Linear nondispersive
	$\square\ O(\mu^{-3})$	Linear dispersive
$\square\ O(1)$	$(\varepsilon\mu)^{-3} \;\square\; \tau \;\square\; \mu^{-3}$	Linear dispersive
	$\square\ O(\varepsilon\mu)^{-3}$	Nonlinear dispersive

Note that the importance of the nonlinearity could be over-estimated Because of the 1D analyses.

Example 1: For Chilean tsunami $h = 4$ km, $A = 6$ m, $\lambda = 200$ km,

$$\varepsilon = 1.5\times10^{-3}, \mu^2 = 4\times10^{-4}, U_r = 3.75.$$

Thus, $t \sim 0.66\times10^6$ sec $= 183$hr and $x \sim 1.32\times10^5$km.

Example 2: For Algerian tsunami $h = 2$ km, $A = 1$ m, $\lambda = 20$ km,

$$\varepsilon = 2\times10^{-3}, \mu^2 = 10^{-2}, U_r = 0.2.$$

Thus, $t \sim \sqrt{2}\times10^4$ sec $= 3.93$hr and $x \sim 2\times10^3$km.

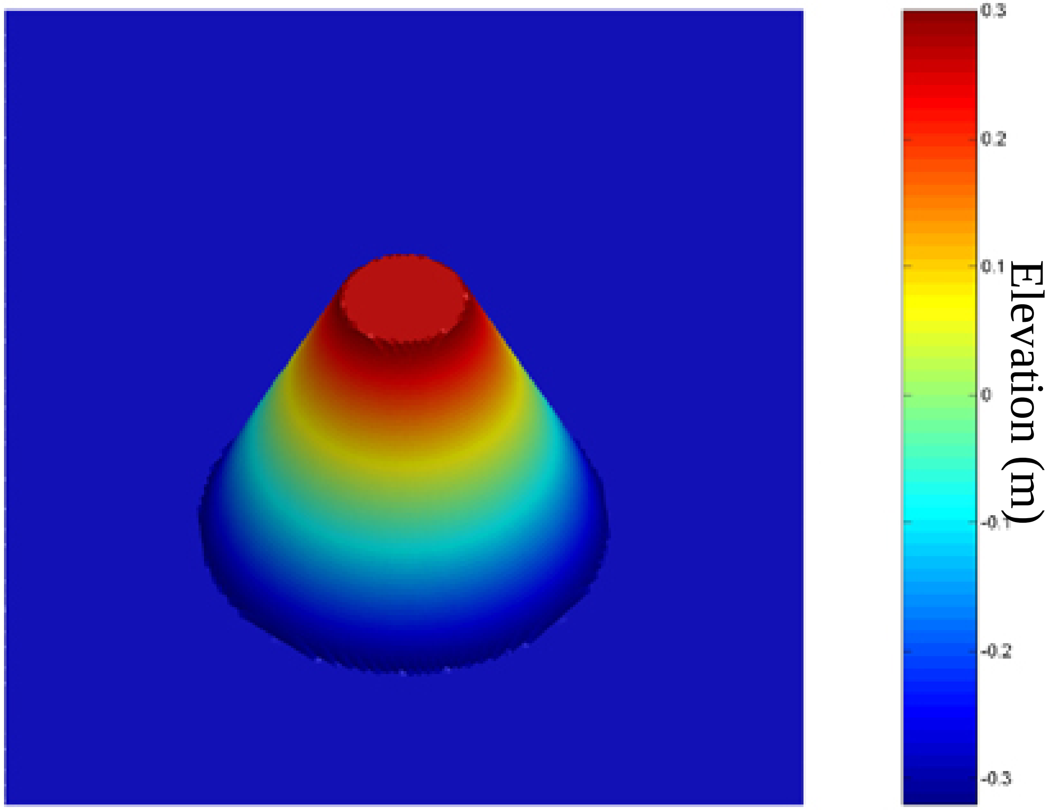
Validation of Runup Algorithm

(Lynett, Wu and Liu 2002, Coastal Engineering)

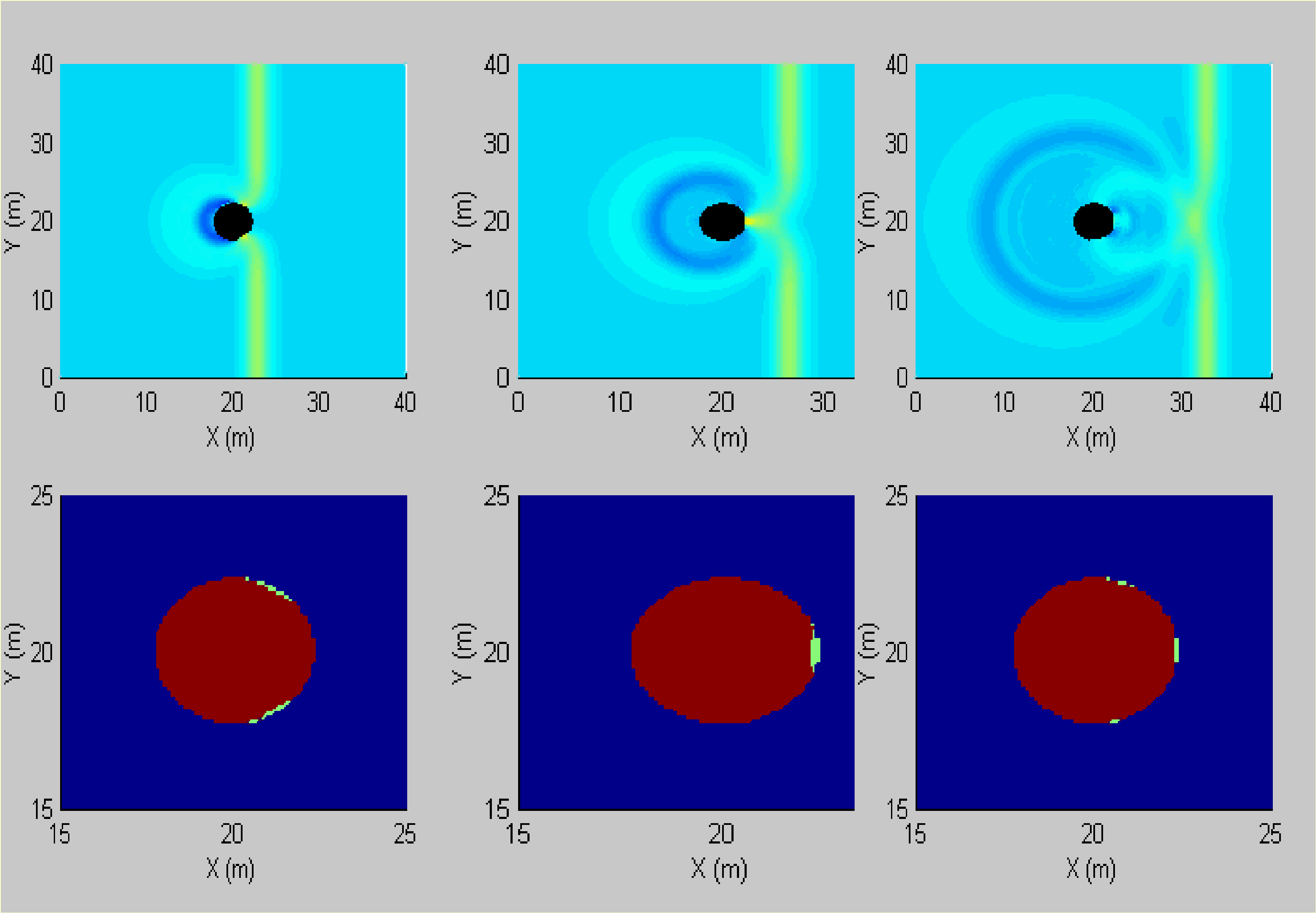
- Runup of solitary wave around a circular island
  - Experimental data taken from Liu *et al.* (*J.F.M.* 1995)

- Physical setup:
  - Still water depth = 0.32 m
  - Slope of side walls = 1:4
  - Depth profile →

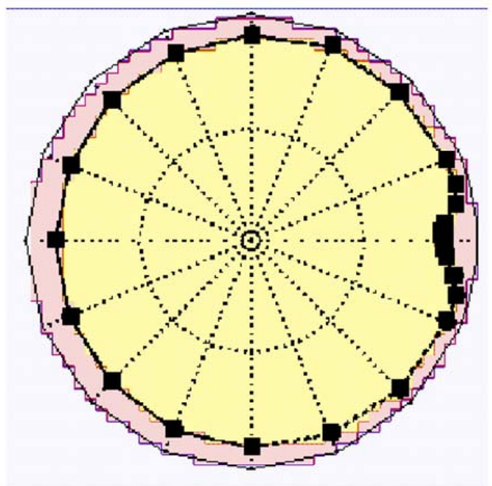
- Numerical simulation of conical island runup:
  - Wave amplitude = 0.028 m
  - Still water depth = 0.32 m
  - Beach slope = 1:4
  - Dx = 0.1 m



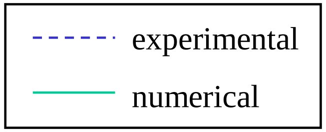
3D animation of runup of solitary wave around a circular island

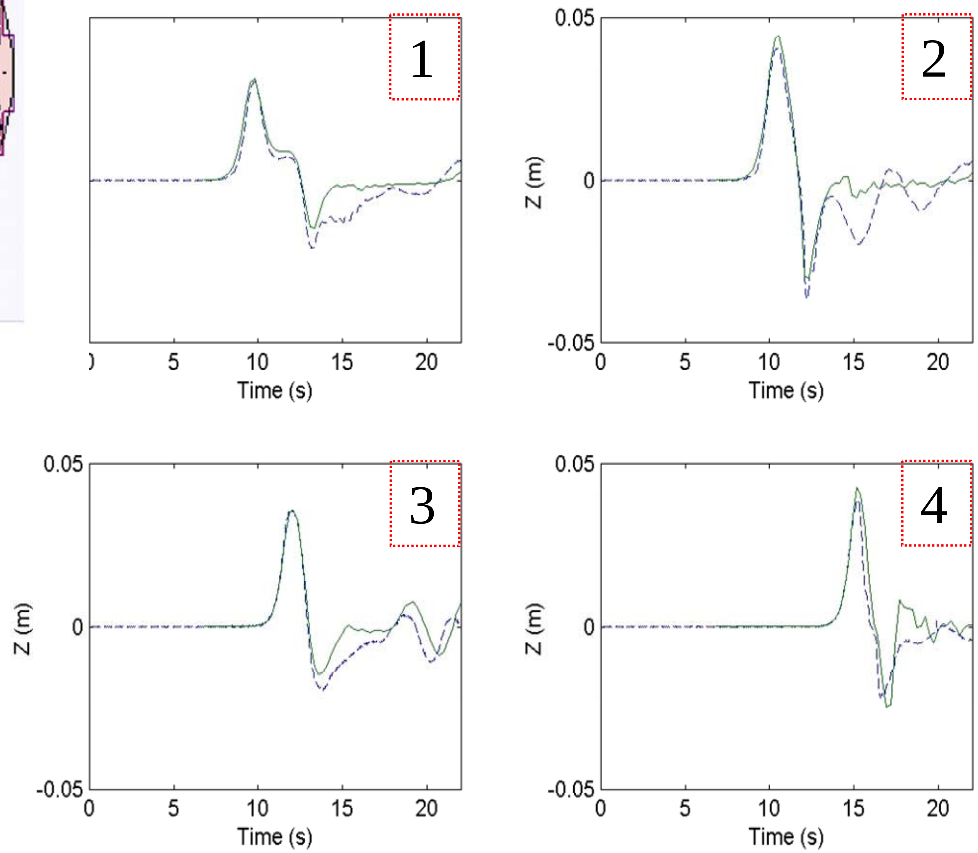


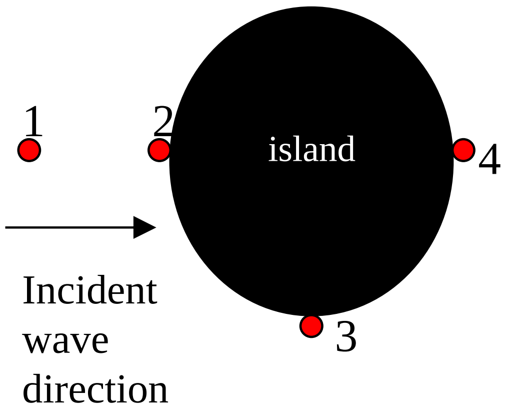
Validation of Runup Algorithm



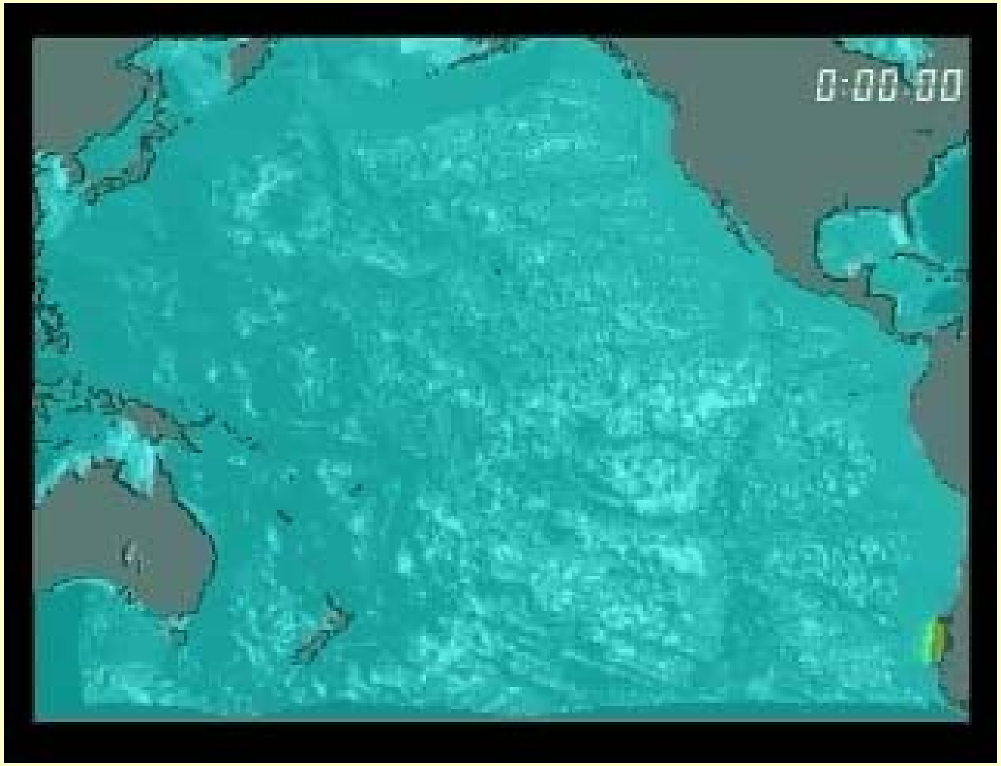
Time series comparisons







1960 Chilean Tsunami



The epicenter of the 1960 Chilean earthquake was located about 100 km offshore of the Chilean coast. The fault zone was roughly 800 km long and 200 km wide, and the displacement of the fault was 24 m. The orientation of the fault was N10 E. The focal depth of the slip was estimated at 53 km with a 90 degree slip angle and a 10 degree dip angle. Using these estimated fault parameters, we can calculate the initial free-surface displacement (Mansinha and Smylie, 1971). The wavelength of the initial tsunami form was roughly 1,000 km and the wave height was roughly 10 m.

1960 Chilean Tsunami Inundation in Hilo Bay

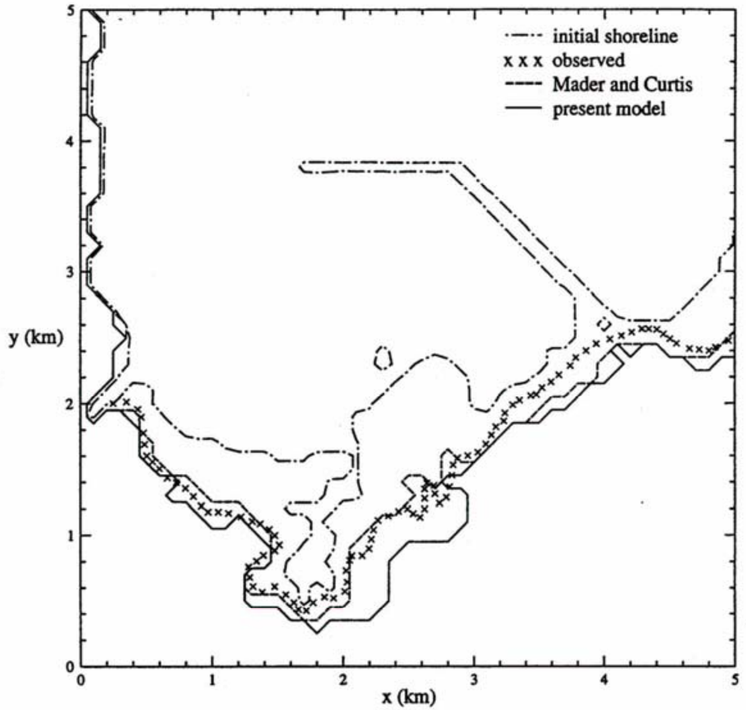


Figure 4-10: The comparison of maximum inundation area.

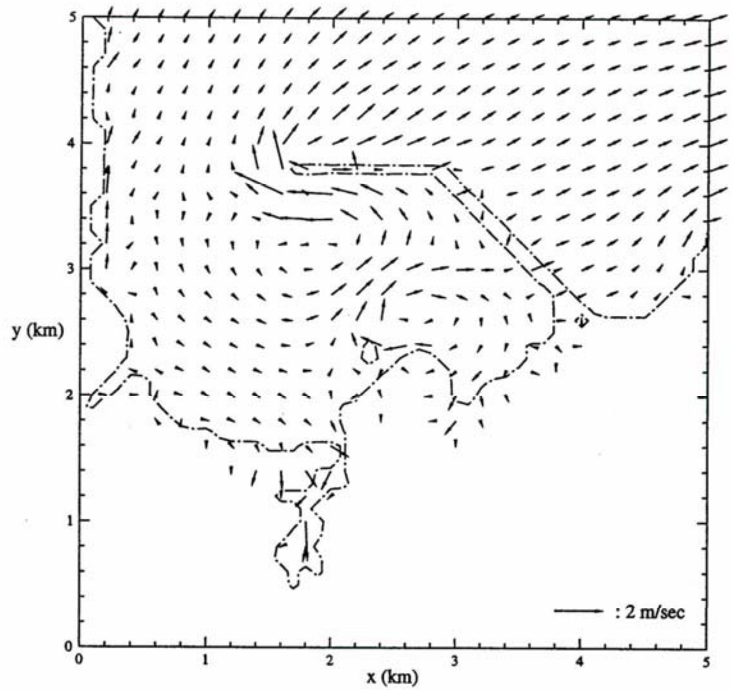


Figure 4-12 (c). The snapshot of velocity distribution (time = 15 hr 20 min).

Some common difficulties in using depth-integrated wave equations

- Most of numerical algorithms are dissipative, especially the moving shoreline algorithms;
- Most of models do not include wave breaking;
- Most of models specify bottom friction coefficients and wave breaking parameters empirically with limited validation;
- Depth-integrated wave equations can not adequately address the wave-structure interaction issues.

Other open issues

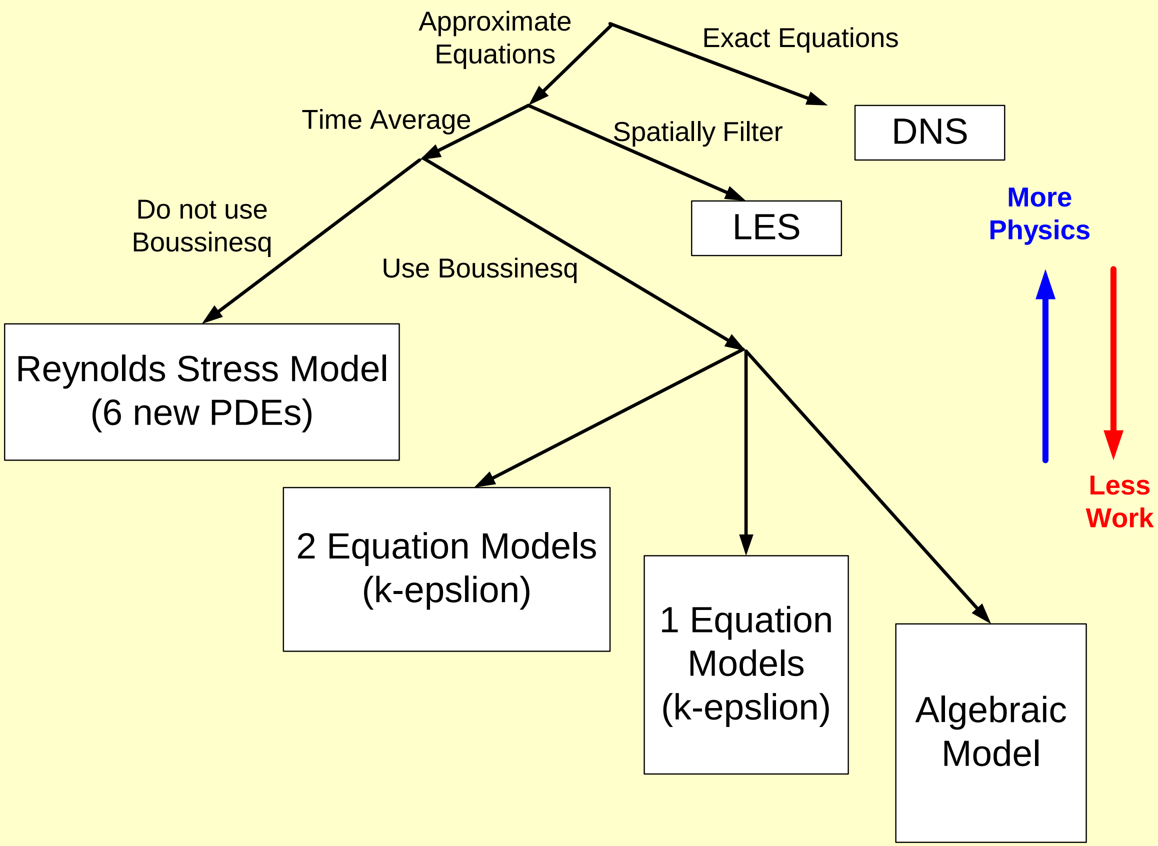
- Coupling the hydrodynamic models with sediment transport models
- Coupling the hydrodynamic models with debris flow models
- Coupling the hydrodynamic models with soil (foundation) dynamic models

3D/2D Numerical Modeling of Tsunamis in Nearshore Environment and Their Interaction with Structures

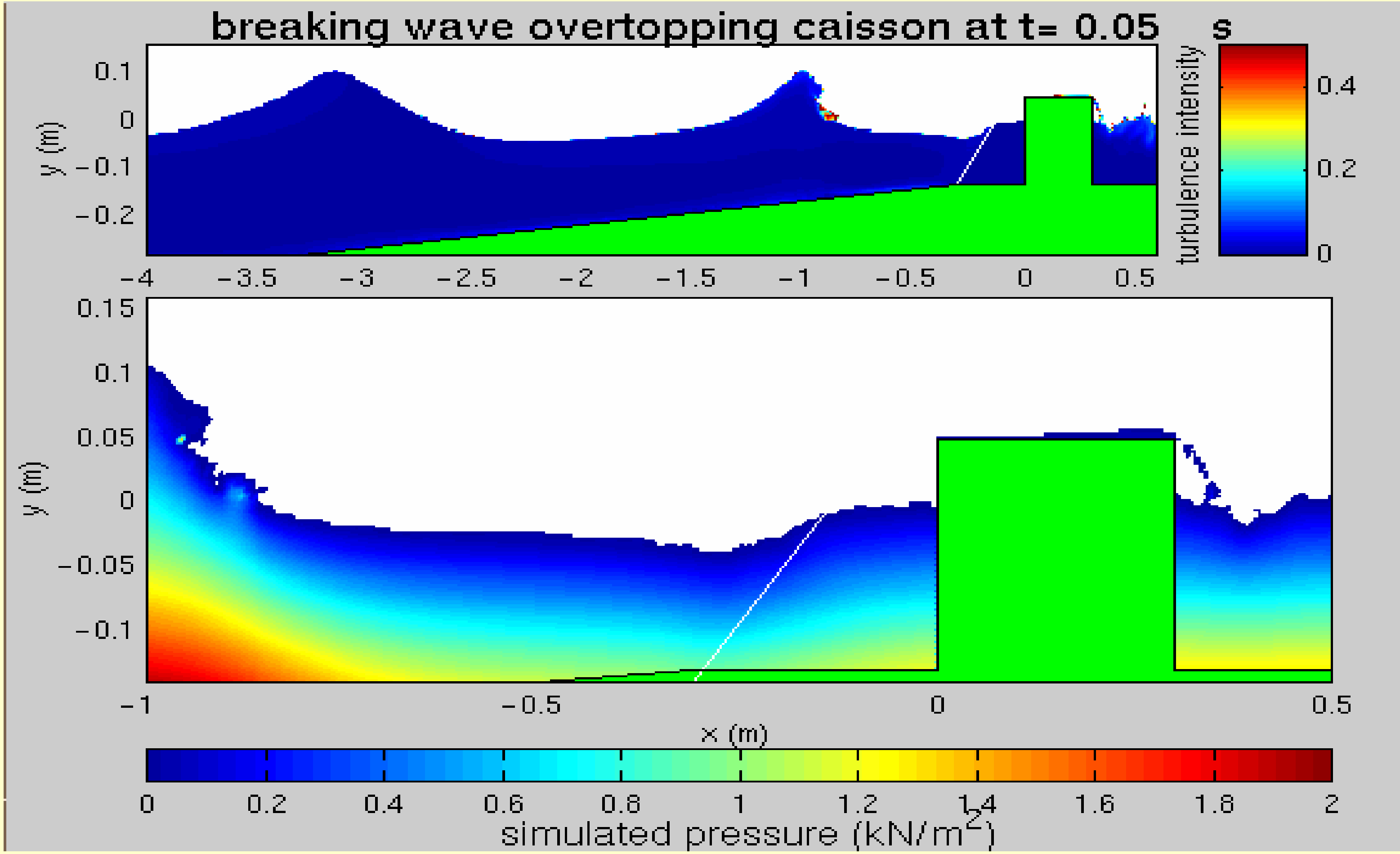
$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla \cdot (\rho\mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \tilde{\tau} + \rho\mathbf{g} \qquad \tilde{\tau} = \mu\left(\nabla\mathbf{u} + \nabla^T\mathbf{u}\right)$$

Turbulence Models



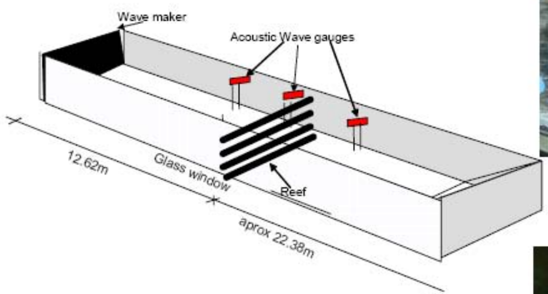
Breaking wave forces and overtopping on a protected caisson breakwater

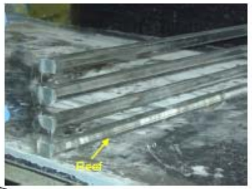


2D animation

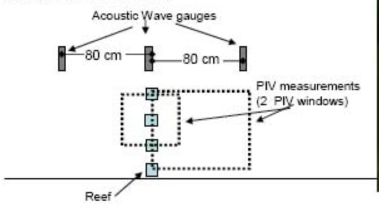
Experiments and Numerical Simulations of Solitary Wave acting on a Submerged-Filter-Reef

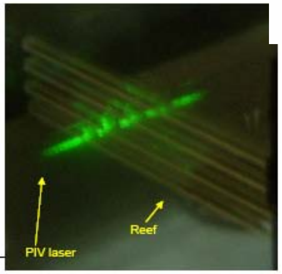
Experimental Set-Up





Measurement Devices





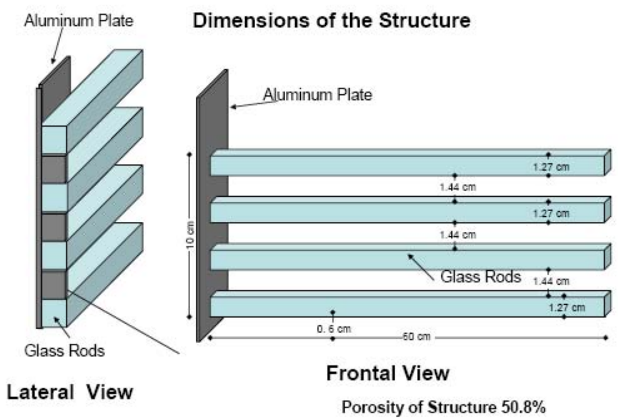


Figure 1: Experimental Set-UP

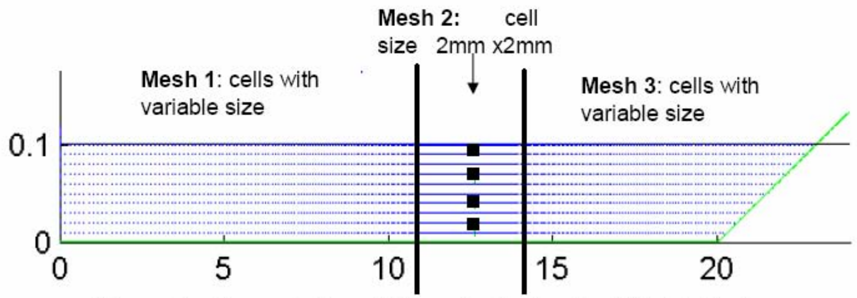
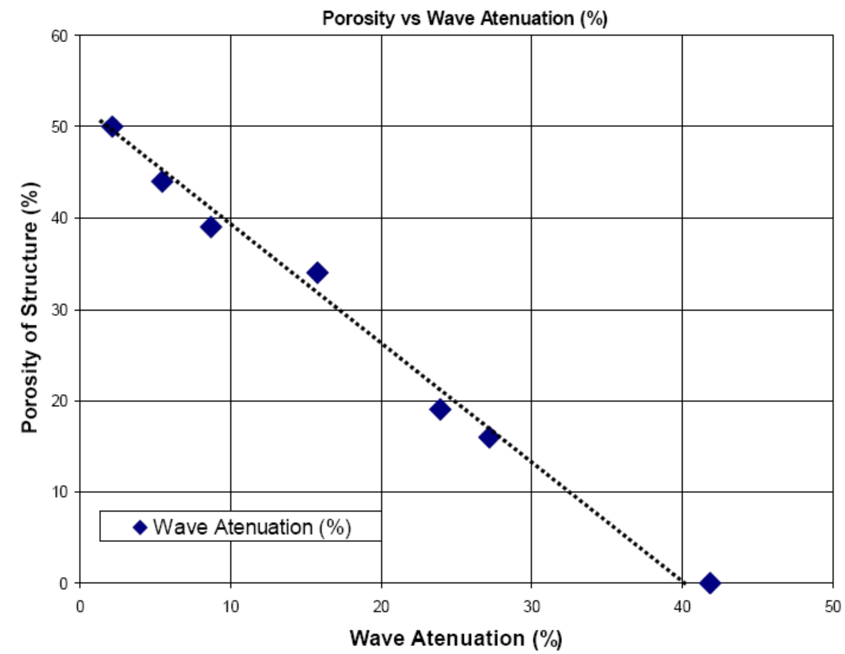
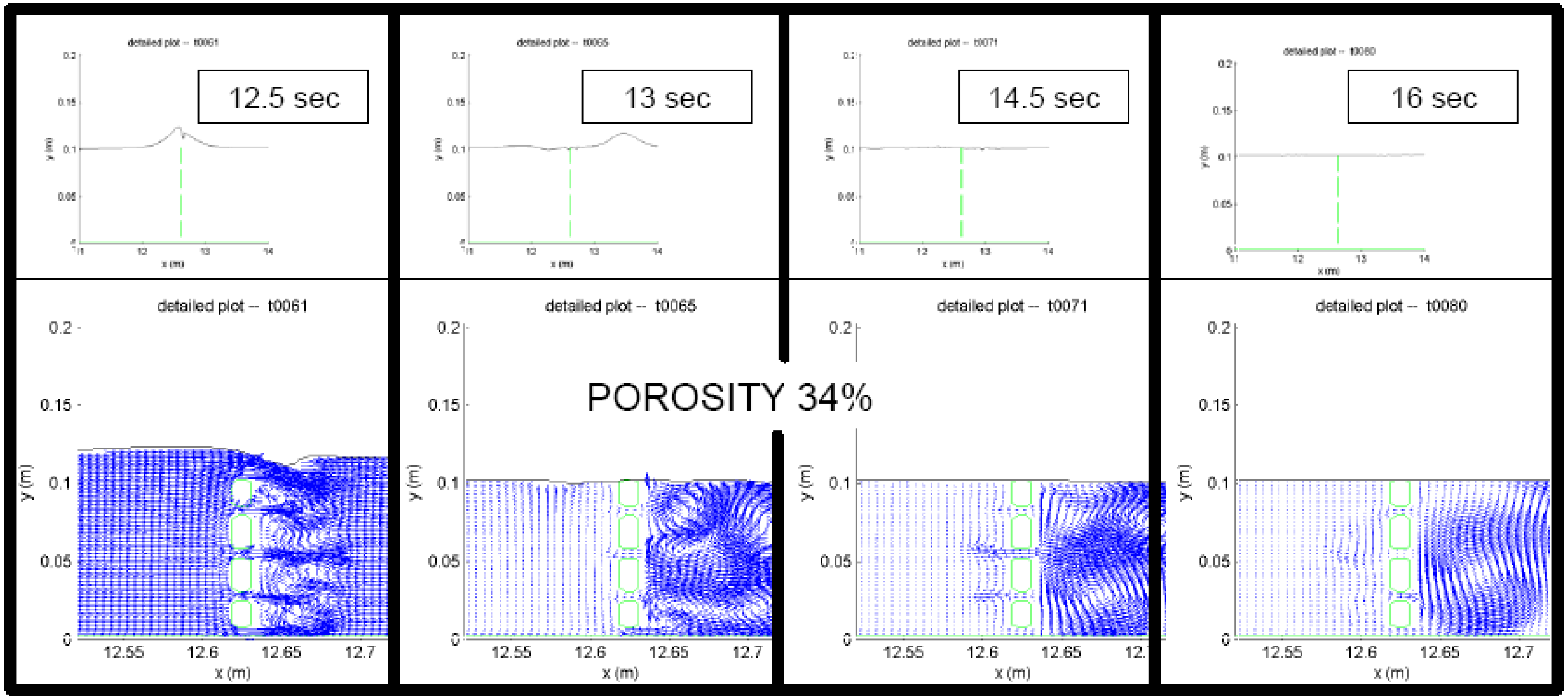
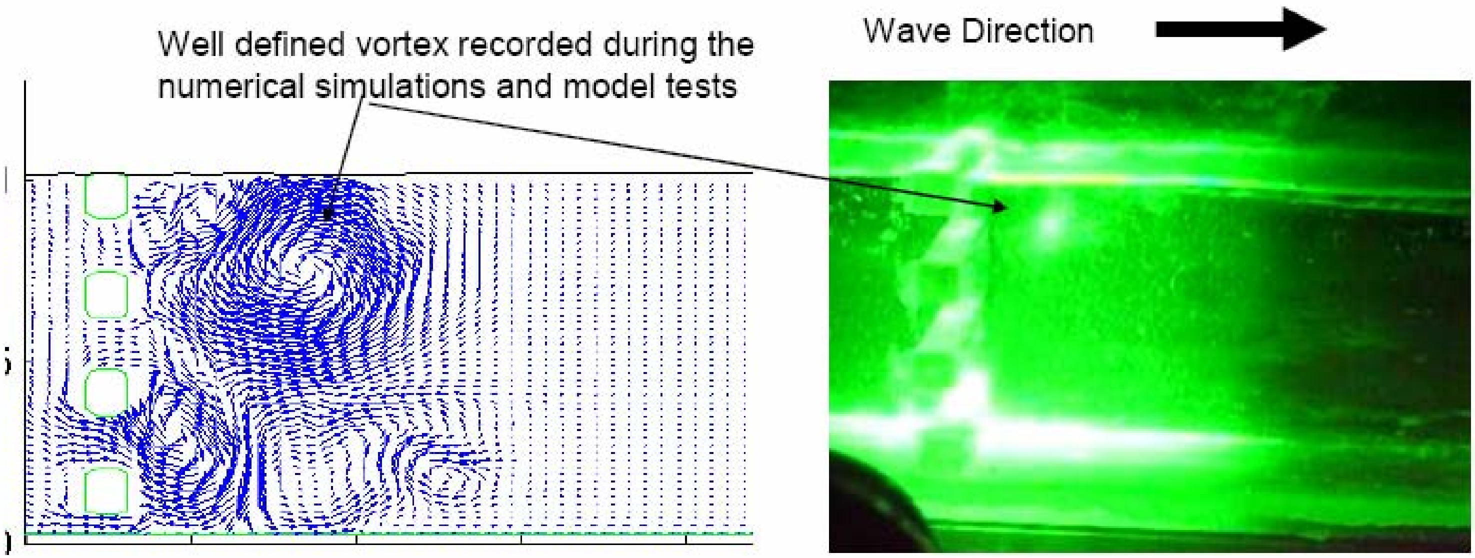


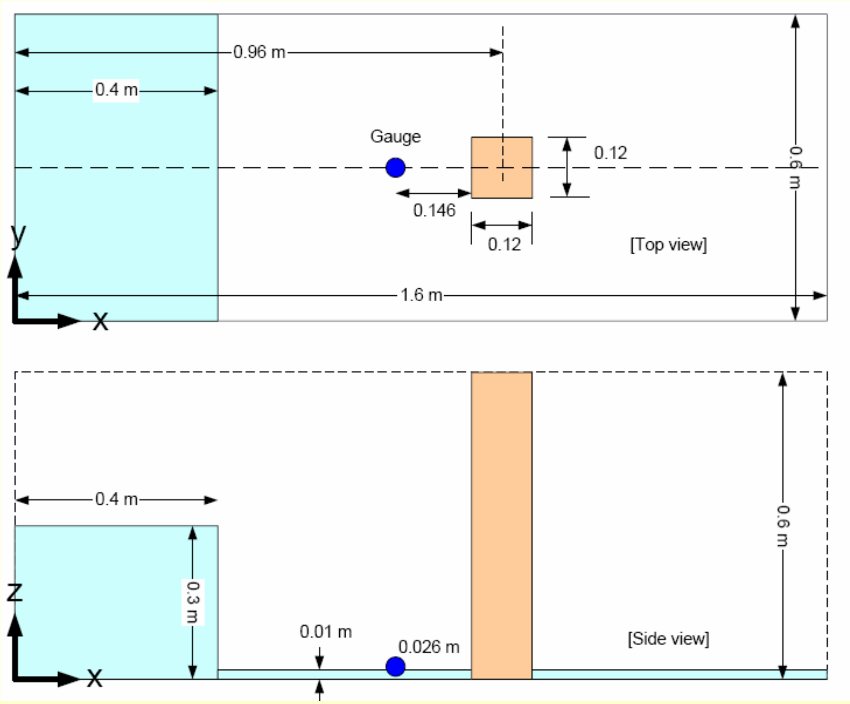
Figure 3: Computational Domain during the Model Tests

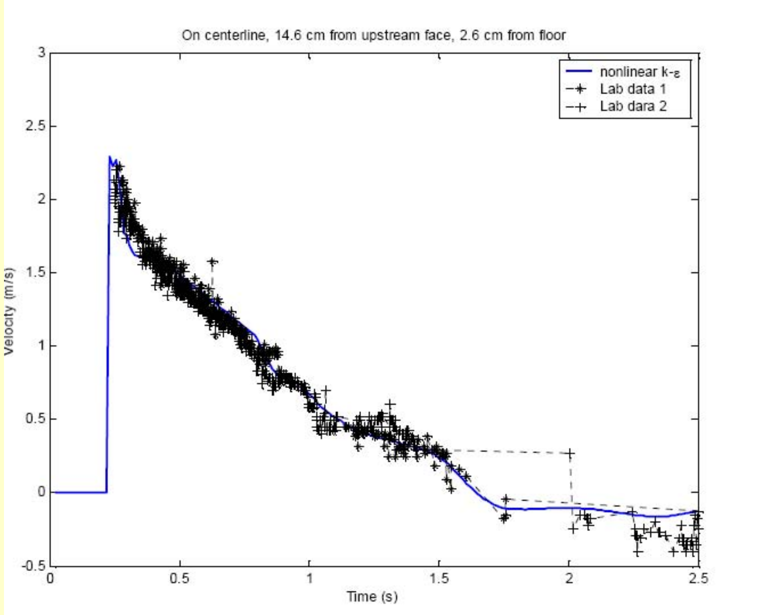


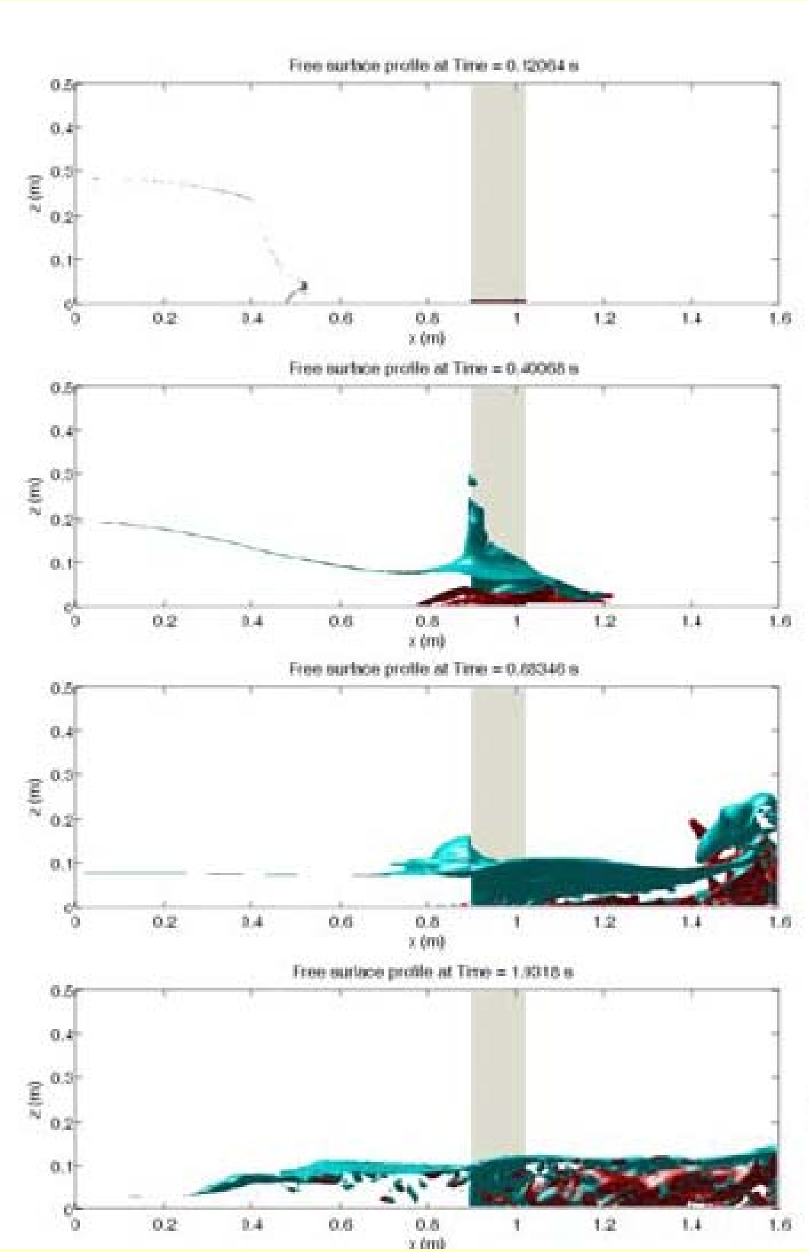

12.5 sec
13 sec
14.5 sec
16 sec
detailed plot -- t0061
detailed plot -- t0065
detailed plot -- t0071
detailed plot -- t0080
POROSITY 34%
y (m)
x (m)


Well defined vortex recorded during the numerical simulations and model tests
Wave Direction

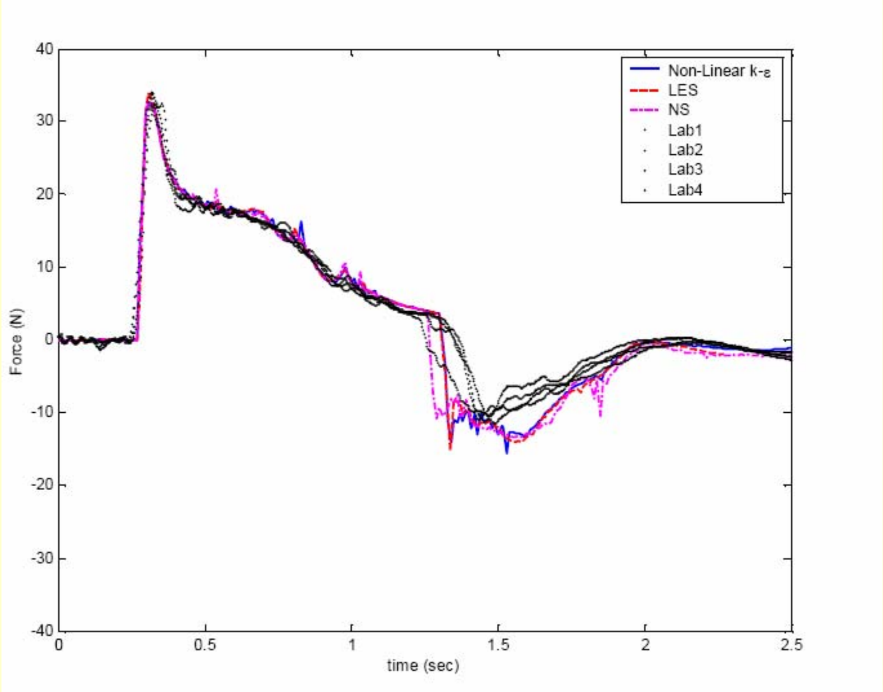
Simulation of a Dam Break Bore interacting with a square cylinder



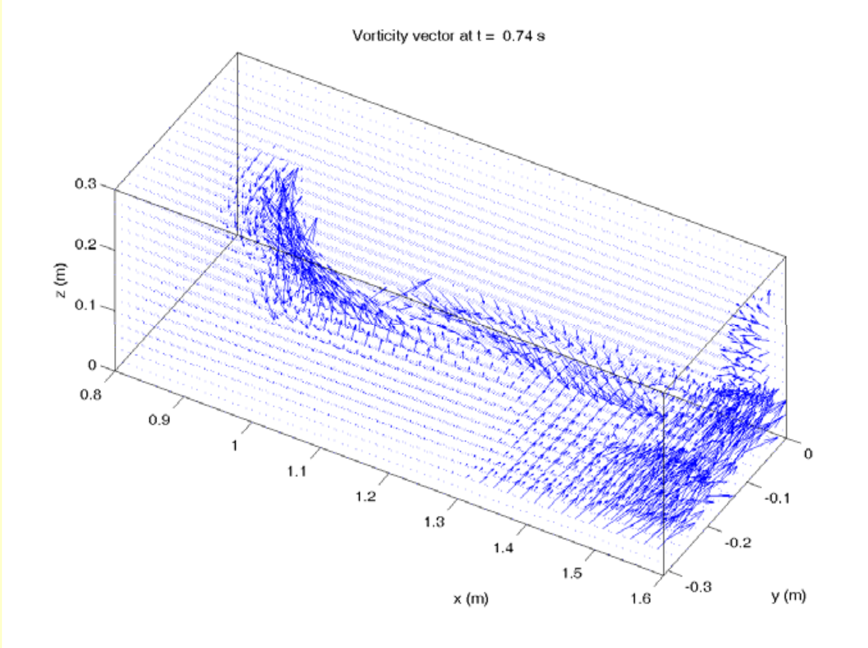




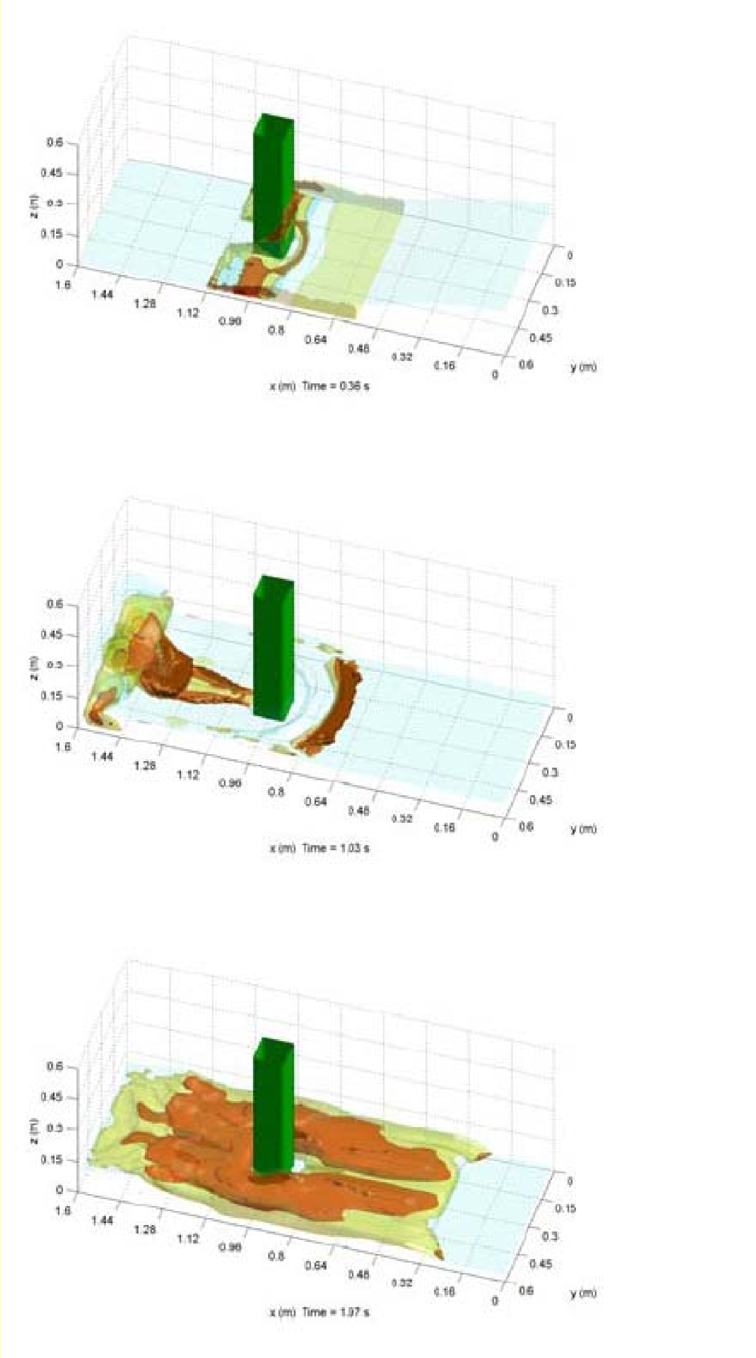
dx = dy = dz =0.02m [Animation VOF]



Horizontal wave force on the cylinder

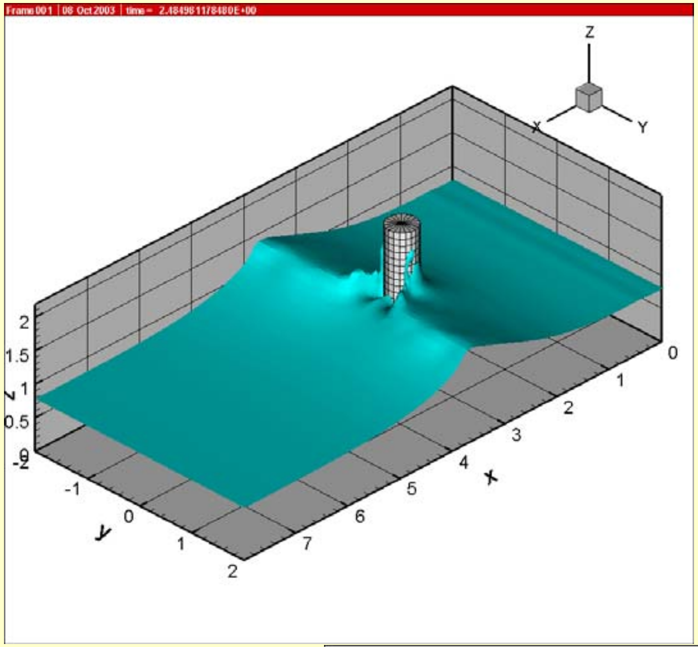


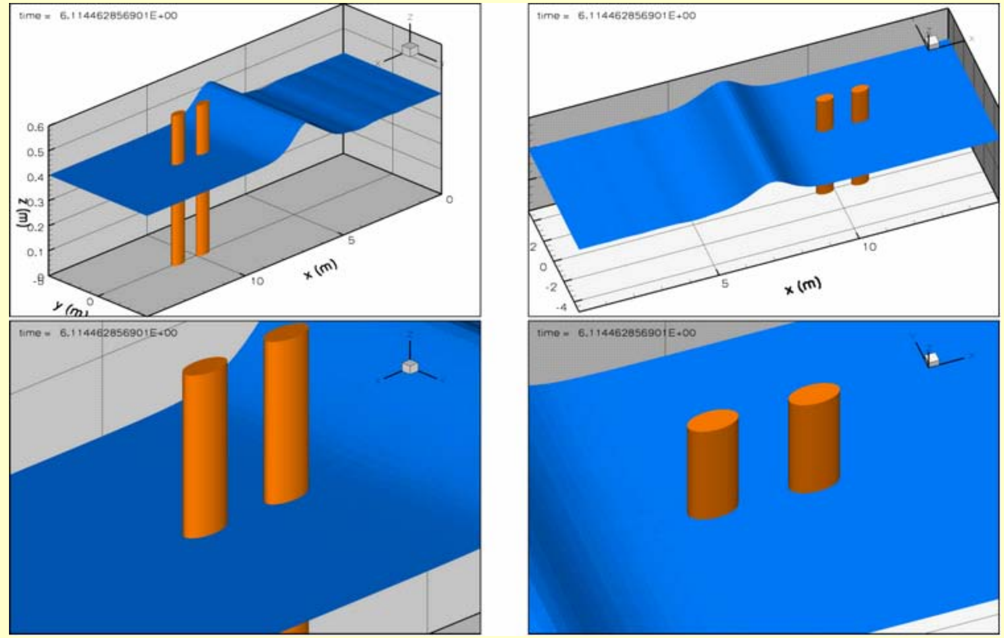
Vorticity field

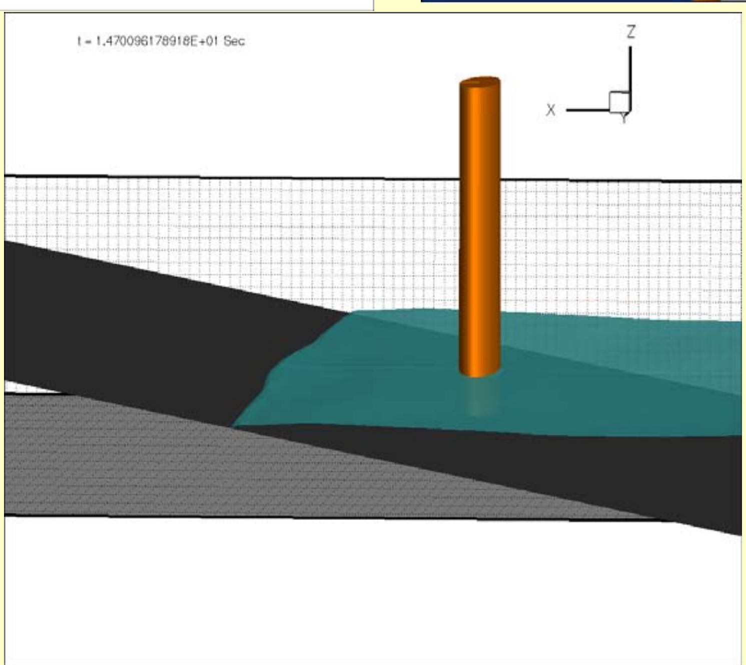


Turbulence kinetic energy

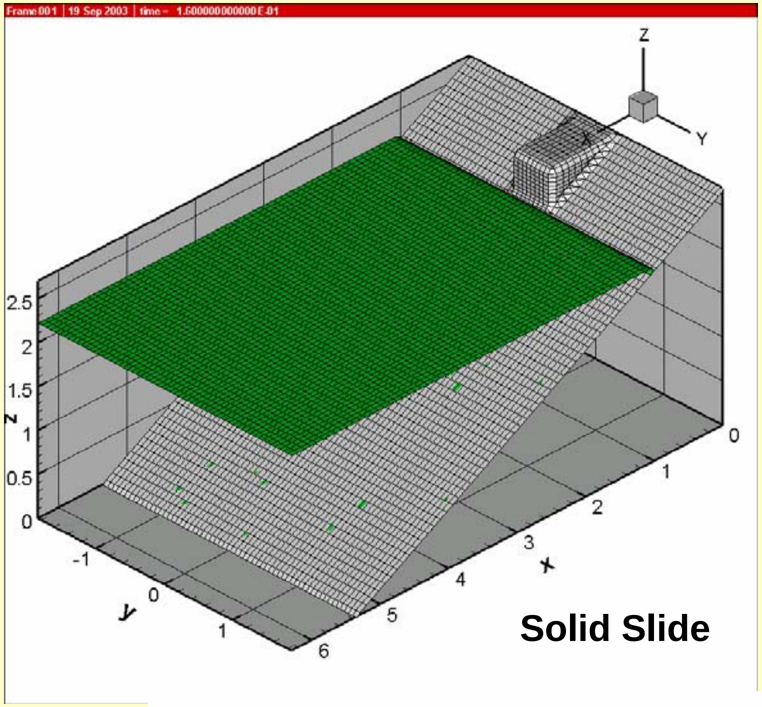
Wave Forces on Single cylinder or a Group of Cylinders



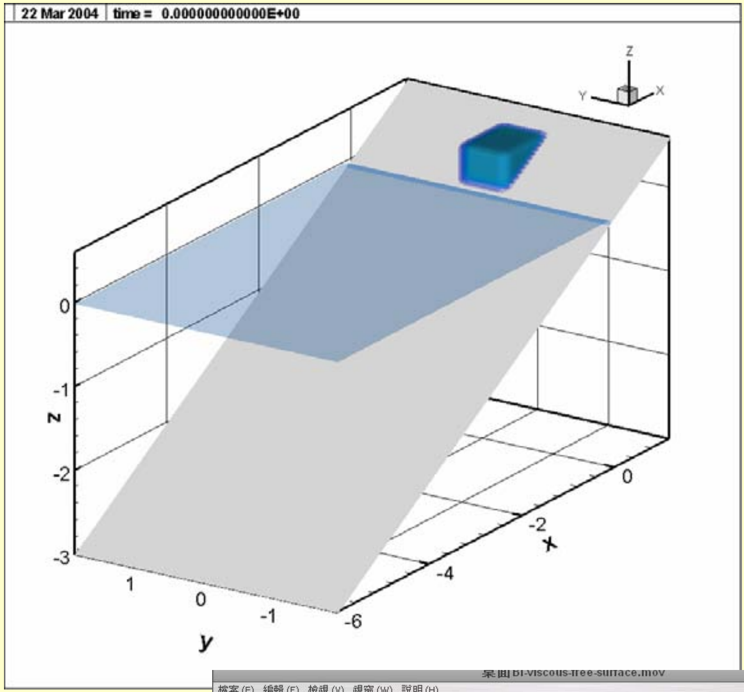




3D Landslide Generated Tsunamis



Solid Slide



Viscous Fluid Slide

$$\alpha = \frac{\nu}{\nu'}$$

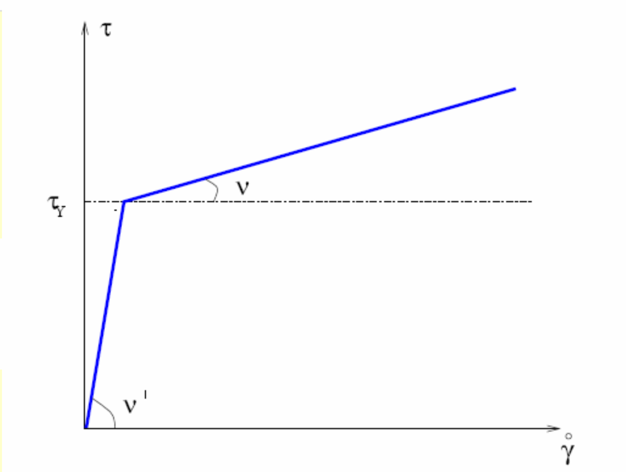
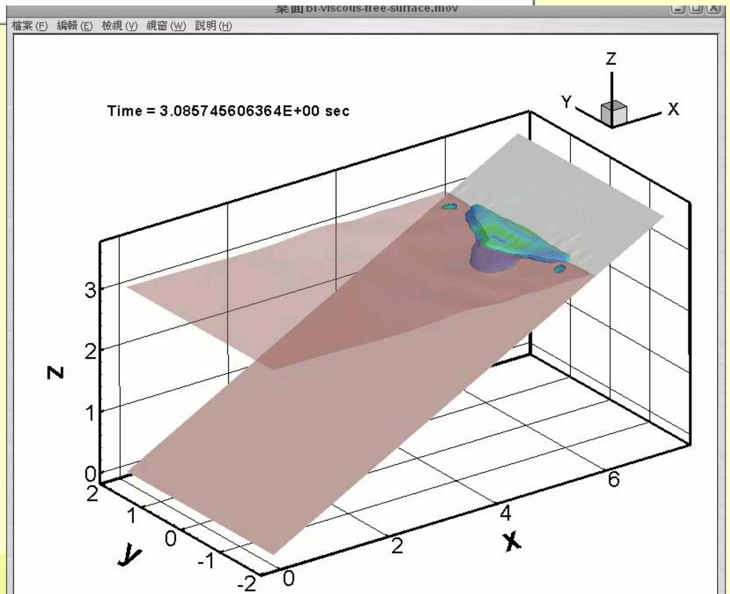


Figure 5: The Rheology of the bi-viscous fluid.



If α goes to zero, the bi-viscous fluid will approaches the Bingham fluid.

If α goes to one, the bi-viscous fluid is reduced to the Newtonian fluid.

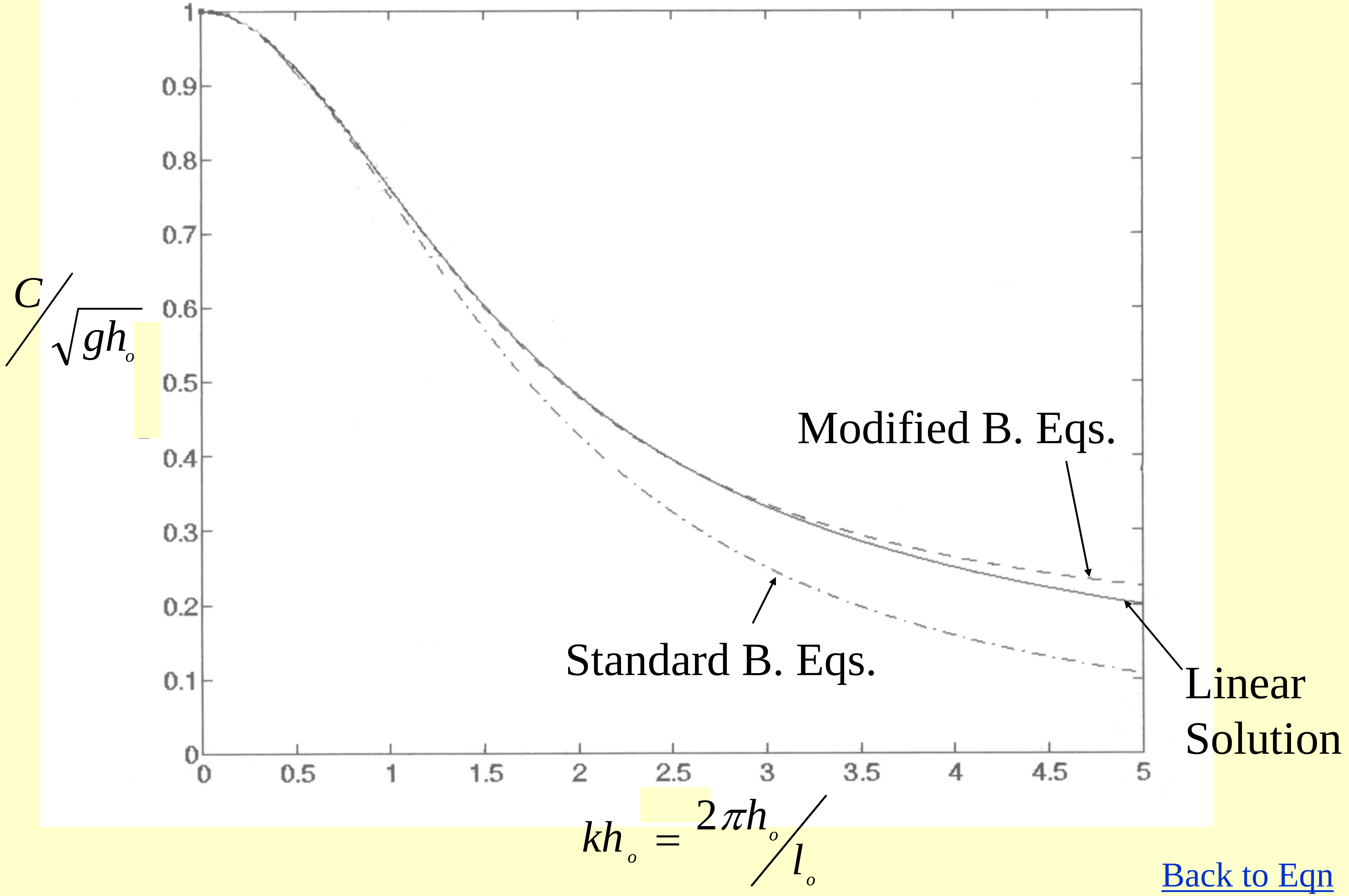
α = 1/100 in current simulations.

τ_p = 0.875 N/m^2 , ρ_m = 1106 kg/m^3, and ν_m = 0.1 N m^{-2}s.

Concluding Remarks

- **For most of distance tsunamis, linear shallow water wave equations are adequate for modeling tsunami propagation in ocean basin. In some cases, linear dispersive wave equations might be required.**
- **In the inner continental shelf region and the nearshore regions, nonlinear shallow water equations might be necessary. However, wave breaking and other dissipative processes need to be better understood and parameterized to be used in the NSWEs.**
- **3D RNS models or LES models are needed to investigate tsunami-structure interactions as well as other nearshore processes, including sediment transport and debris flows.**
- **3D models can be coupled with 2D SWE and NSWE or Boussinesq-type wave equations.**
- **3D results can yield the parameters needed in 2D models.**

Improved Linear Dispersion Relationship



Back to Eqn

Momentum Equation (Dimensionless)

$$u_{\alpha t} + \varepsilon u_{\alpha} \cdot \nabla u_{\alpha} + \nabla \zeta +$$

$$\mu^{2}\left\{\frac{z_{\alpha}^{2}}{2}\nabla\left(\nabla \cdot u_{\alpha t}\right) + z_{\alpha}\nabla Q_{t} + z_{\alpha t}\left[z_{\alpha}\nabla\left(\nabla \cdot u_{\alpha}\right) + z_{\alpha}\nabla Q\right]\right\} +$$

$$\varepsilon\mu^{2}\left[Q\nabla Q - \nabla \zeta Q_{t} + \left(u_{\alpha} \cdot \nabla z_{\alpha}\right)\nabla Q + z_{\alpha}\nabla\left(u_{\alpha} \cdot \nabla Q\right)\right] +$$

$$\varepsilon\mu^{2}\left\{z_{\alpha}\left(u_{\alpha} \cdot \nabla z_{\alpha}\right)\nabla\left(\nabla \cdot u_{\alpha}\right) + \frac{z_{\alpha}^{2}}{2}\nabla\left[u_{\alpha} \cdot \nabla\left(\nabla \cdot u_{\alpha}\right)\right]\right\} +$$

$$\varepsilon^{2}\mu^{2}\nabla\left\{\frac{\zeta^{2}}{2}\nabla \cdot u_{\alpha t} - \zeta u_{\alpha} \cdot \nabla Q + \zeta Q \nabla \cdot u_{\alpha}\right\} +$$

$$\varepsilon^{3}\mu^{2}\nabla\left\{\frac{\zeta^{2}}{2}\left[\left(\nabla \cdot u_{\alpha}\right)^{2} - u_{\alpha} \cdot \nabla\left(\nabla \cdot u_{\alpha}\right)\right]\right\} = O\left(\mu^{4}\right)$$

where $: \; Q = \nabla \cdot \left(h u_{\alpha}\right) + \dfrac{h_{t}}{\varepsilon}$

Back to
Main Slide

Sediment deposit by tsunamis

2002 PNG tsunami

1998 PNG tsunami (The scale is 8cm long)