THEORETICAL FORMULAE FOR WAVE SLAMMING LOADS ON SLENDER CIRCULAR CYLINDERS AND APPLICATION FOR SUPPORT STRUCTURES OF WIND TURBINES

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Based on large scale experiments theoretical formulae for the load due to breaking wave impact on slender piles have been developed. The load distribution in time and space is given by the proposed theoretical 3D-impact model. The formulae are applied to calculate the impact force on a support tower of a wind turbine subjected to breaking wave. The bending moment at the base of the support tower can also be determined.

1. Introduction

Many offshore wind turbines are planned at the German coast of the North and Baltic sea. Usually these plants will be supported by mono-piles. In this case, the wave load is generally predicted by the Morison equation.

The calculated values will be exceeded if the waves break just in front of the tower. Therefore, SPM 1984 recommends a factor of 2.5 for breaking waves. However this procedure is inadequate since the dynamic response of the towers must be considered. For this purpose information about the time history of the impact load is required.

Goda (1966) proposed a model for the impact force by considering the breaking wave as a vertical wall of water hitting the cylinder with wave celerity:

$$F(t) = \lambda \cdot \eta_b \cdot \pi \cdot \rho \cdot R \cdot C^2 \cdot (1 - C/R \cdot t) \qquad (1)$$

where C is the wave celerity, R the radius of the cylinder, η_b the maximum elevation of the breaking wave, λ the curling factor and ρ the density of water (Figure 1).

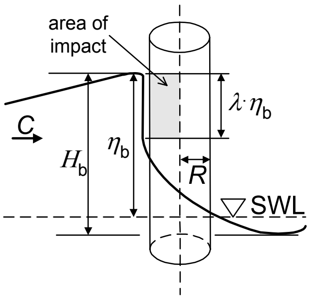


Figure 1. Definition Sketch.

Basco and Niedzwecki (1989) published experimental results indicating much higher impact forces than those predicted by the Goda model. There is a considerable uncertainty concerning the estimation of the breaking wave loads (Matthies et al. 1995).

Large-scale experiments were carried out in the Large Wave Channel of the Coastal Research Centre in Hannover, Germany, a joint institution of both Hannover and Braunschweig Universities. The experimental results were used to develop an analytical description for the time history of the slamming force on a cylinder due to breaking wave attack.

2. Calculation of the Slamming Force

2.1. *Maximum Line Force*

The line force is maximum at the beginning of the impact (Wienke and Oumeraci 2004):

$$f = 2 \cdot \pi \cdot \rho \cdot R \cdot C^2 \tag{2}$$

This maximum line force corresponds to a slamming coefficient of 2π which is twice the value assumed for the model of Goda (Wienke et al. 2000).

2.2. *Time History*

The time history of the slamming force is shown in Figure 2. The corresponding duration of the impact is given by the following equation (Wienke 2001; Wienke and Oumeraci 2004):

$$T = \frac{13}{32} \cdot \frac{R}{C} \tag{3}$$

This duration is less than half the time given by the model of Goda. The decrease of the impact force with time is steeper than proposed by Goda since the duration is shorter and the maximum intensity is higher.

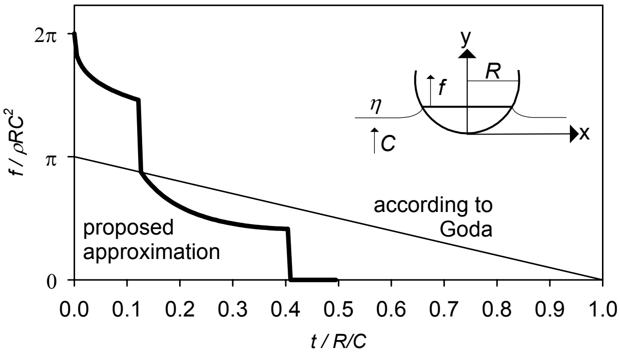


Figure 2. Time History of Impact Force.

2.3. *Slamming Force*

The impact of a breaking wave on a vertical cylinder is shown in Figure 3. The impact starts at one point in the front line of the cylinder and spreads out radially and tangentially to the surface of the cylinder. Thus, the impact takes place almost simultaneously along the height of the cylinder. Therefore, the time history of the impact force is equal to the time history of the line force multiplied with a factor having a length dimension. This factor is given by the maximum elevation of the breaking wave η_b and the curling factor λ which is experimentally determined to be $\lambda \approx 0.5$ (Wienke et al. 2001).

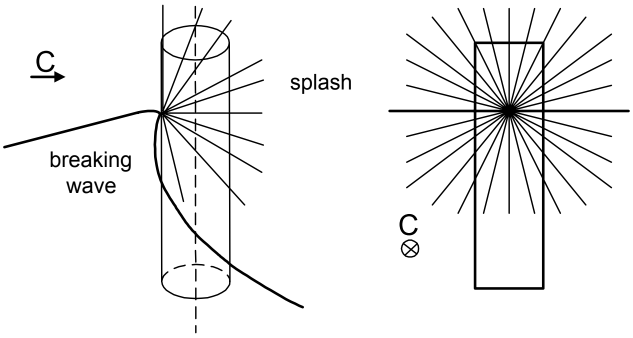


Figure 3. Splash Indicating the Spreading of Impact.

2.4. *Vertical Distribution of the Line Force*

In a useful approximation a constant line force along the height $\lambda\cdot\eta_b$ can be assumed. This area of impact extends immediately below the wave crest as sketched in Figure 1. In this way an upper limit is calculated for the force and overturning moment at the base of the cylinder.

2.5. *Formulae for Normal Impact*

Summing up, the three-dimensional description of the impact force F normal to the cylinder axis is given as follows (Wienke 2001; Wienke and Oumeraci 2004):

$$F = \lambda \cdot \eta_b \cdot \rho \cdot R \cdot C^2 \cdot \left(2 \cdot \pi - 2 \cdot \sqrt{\frac{C}{R} \cdot t} \cdot \mathrm{Artanh} \sqrt{1 - \frac{1}{4} \cdot \frac{C}{R} \cdot t} \right) \qquad (4)$$

$$\text{for} \quad 0 \le t \le \frac{1}{8} \cdot \frac{R}{C}$$

$$F = \lambda \cdot \eta_b \cdot \rho \cdot R \cdot C^2 \cdot \left(\pi \sqrt{\frac{1}{6} \cdot \frac{1}{\frac{C}{R} \cdot t'}} - \sqrt[4]{\frac{8}{3} \cdot \frac{C}{R} \cdot t'} \cdot \mathrm{Artanh} \sqrt{1 - \frac{C}{R} \cdot t' \cdot \sqrt{6 \cdot \frac{C}{R} \cdot t'}} \right) \quad (5)$$

$$\text{for} \quad \frac{3}{32} \cdot \frac{R}{C} \le t' \le \frac{12}{32} \cdot \frac{R}{C} \quad \text{with} \quad t' = t - \frac{1}{32} \cdot \frac{R}{C}$$

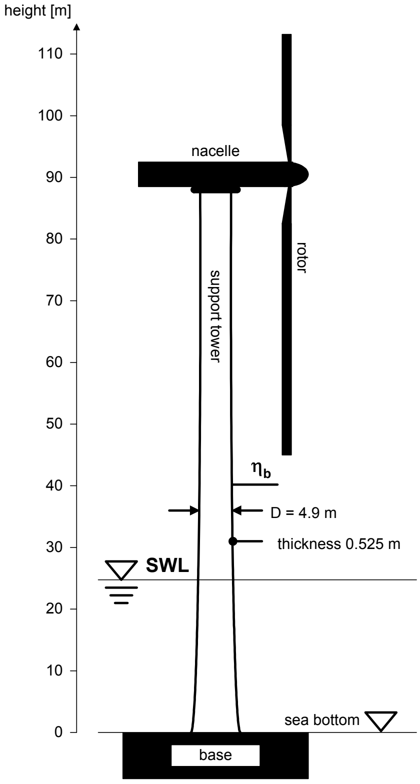
The curling factor λ is equal to 0.5 for plunging breakers which cause the highest impact force and thus the most relevant extreme loading.

3. Example Application

Slamming loads due to breaking waves are calculated for the monopile support structure of an offshore wind turbine. The example is taken from a study on offshore wind energy in Europe (Matthies et al. 1995).

The considered offshore wind turbine is a British plant with a power of 6 MW (Figure 4). The rotor consists of 2 blades and has a diameter of 100 m. The nacelle is mounted on a concrete tower in a height of 88 m. The support tower has a diameter of 4.9 m and a thickness of 0.525 m. It is placed on a base plate.

The water depth at the plant is 25 m. This water depth is in the range of water the depth conditions of the offshore wind parks which are planned in Germany. For example, west off the island Sylt the wind park Butendiek, which construction will start 2006, is planned in a water depth of 20-30 m, using monopile support towers with a diameter of about 5 m.



Characteristic data

Wind turbine	
Power	6 MW
Number of blades	2
Diameter of rotor	100 m
Support tower	
Material	Concrete
Length	88 m
Diameter	4.9 m
Thickness	0.525 m
Frequency (1st order)	0.3473 Hz
Frequency (2nd order)	1.941 Hz
Ambient conditions	
Water depth	25 m
Breaking wave parameters	
Type	Plunging
Period	9 s
Celerity	12.45 m/s
Height	16 m
Water surface elevation	15.2 m
Horizontal asymmetry	0.95

Figure 4. Sketch of the Offshore Wind Turbine Selected for the Example Application.

The natural frequencies of the support tower are 0.3473 Hz for the 1st order and 1.941 Hz for the 2nd order. The load at the base is dominated by these two oscillation frequencies. Higher orders can be neglected when the moment at the base is determined. Due to the shape of vibration these higher order oscillations must only be considered for different locations along the height of the support tower. The maximum overturning moment acts at the base (Figure 4).

The breaking wave has a period of 9 s. This period results in a wave celerity of 12.45 m/s when linear wave theory is applied. The breaking wave height is about $H_b \approx 16$ m.

The impact force depends on the surface elevation η_b of the breaking wave above the still water level. This quantity follows from the horizontal asymmetry $\mu = \eta_b / H_b$ of the breaking wave. The maximum value of the horizontal asymmetry is 0.95. This value was determined by field measurements (Myrhaug and Kjeldsen 1986). For a conservative calculation the maximum water surface elevation is set to 15.2 m. A plunging breaker is assumed so that the curling factor is 0.5.

Using Eq. 2 with a radius of the monopile $R = 2.45$ m and a wave celerity $C = 12.45$ m/s a maximum line force of $f_{max} = 2386$ kN/m is obtained. According to Eq. 3 the corresponding duration of the impact is $T \approx 80$ ms. The time history of the line force which is calculated with Eqs. 4 and 5 by setting $\lambda \cdot \eta_b = 1$ is plotted in Figure 5.

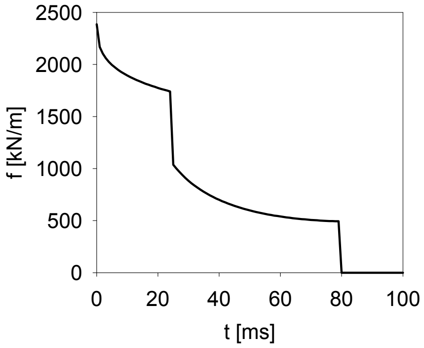
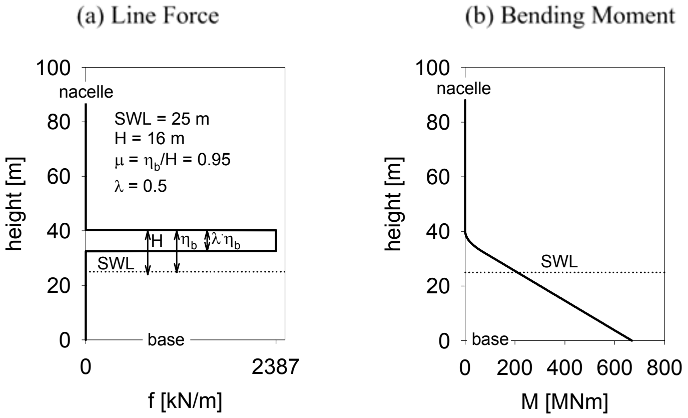


Figure 5. Time History of the Impact Force.

The impact occurs simultaneously at each level of the support tower. Therefore, the time history of the impact force can be considered as identical with the time history of the line force. To get the impact force, the line force is multiplied by the time invariant impact height $\lambda \cdot \eta_b = 7.6$ m. This leads to a rectangular shape of the impact area. Covering the height between wave crest and half elevation above still water level, such an impact area represents rather a conservative model.

The distribution of the maximum line force along the height of the support tower is shown in Figure 6a. The resulting bending moment along the height of the cylinder is plotted in Figure 6b. The maximum acting momentum at the base is 668.8 MNm.

In comparison with a non-breaking wave of a corresponding height and period the resulting maximum moment at the base is 110 MNm. The same moment acts as a quasistatic component of the total moment induced a breaking wave on the monopile.


(a) Line Force

(b) Bending Moment

Figure 6. (a) Distribution of Impact Line Force Along the Height and (b) Corresponding Bending Moment.

Due to the short duration of the slamming force, the response of the support tower is small compared with the applied load. Considering the first and second order of the tower oscillation the maximum overturning moment at the base is 16 MNm. The corresponding time history of the bending momentum is plotted in Figure 7.

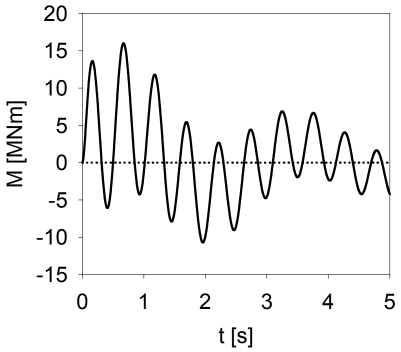


Figure 7. Response at the Base of the Offshore Wind Turbine.

Averaging the impact force over the period of the wave a mean line force of 9.1 kN/m is calculated. If this static force is used to determine the bending moment at the base a value of 2.5 MNm is yielded. Obviously, this static force represents an inadequate substitution of the impact force.

4. Conclusions

A new theoretical model of the time dependent impact force on slender vertical cylinders due to wave breaking has been developed. The most important characteristics of the new model are:

- *Pile-up effect* is taken into account: the maximum force becomes larger and the impact duration is reduced accordingly.
- *Radial spreading* is considered: The impact takes place simultaneously at the different levels of the pile.

The model is described by the analytical formulae given in Eqs. 4 and 5. The applicability of the model is illustrated by an example: The impact force on a support tower of a wind turbine has been calculated, showing that the applied overturning moment at the base is strongly damped by the response of the structure. Not yet well understood is, however, the dynamic response of the soil around the pile to the high frequency motions caused by the breaking wave impact. Whether and how much residual pore pressure will be induced, as this is the case beneath a gravity structure, is still unknown (Kudella and Oumeraci, 2004).

With the new model a comparison between the characteristic periods of the tower oscillation and the duration of a breaking wave impact is made possible.

Acknowledgments

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