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COMPARATIVE ANALYSIS OF WAVE TRANSMISSION OVER AN ARITICIFIAL REEF USING FFT, WAVELET AND HILBERT-HUANG-TRANSFORM

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Abstract: When waves in shallow water, where nonlinear effects predominate, interact with structures, they change their shape, energy and behaviour. To understand the processes associated with this interaction, it is necessary to analyse the wave parameters before and after this interaction. To investigate the influence of the wave analysis methods used on the results, hydraulic model tests with regular waves and wave spectra over an artificial reef with a simple geometry, were comparatively analysed using Fast Fourier Transform (FFT), Wavelet transform (WT) and Hilbert-Huang-Transform (HHT). The results clearly show that none of these methods is able to provide a reliable description of the nonlinear processes associated with the wave transformation at structures in shallow water.

INTRODUCTION

The study of insufficiently known processes such as those associated with wave-structure interaction in shallow water very often requires the performance of well-controlled laboratory experiments resulting in a considerable amount of data. The efficient pre-processing and analysis of these data, which are generally time series, is a prerequisite to improve our understanding of the underlying processes and to increase our ability to predict them.

Seemingly due to the greater difficulties of our brain to process in the frequency domain than in the time domain, Fourier analysis (FT) was the first and still the most widely used method of analysis of time series. It primarily complements the purely temporal description of the signals. Fourier analysis is relatively simple, has several advantages and represents the most known and verified method, due to its extensive application and the associated developments of algorithms and techniques for

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frequency analysis. However, frequency analysis was basically developed for stationary processes, while most of the real signals, particularly those related to sea-waves, are definitively non-stationary. In fact, the frequency content of most signals in the real coastal engineering world evolves over time and the ordinary power spectrum does not reveal such information.

In order to overcome this problem, several alternatives for a joint time-frequency analysis have been developed and widely investigated.

Particularly the last decade has witnessed an impressive increase of the use of these methods in the engineering community. Among these relatively new methods, the Wavelet Transform (WT) and the Hilbert-Huang-Transform (HHT) have become the most popular in coastal and ocean engineering in the last years.

Although these joint time-frequency analysis methods offer a number of advantages in comparison to Fast Fourier Transform (FFT) they are, like FFT, linear analysis methods; i.e. the basic components (sine and cosine waves for FFT, mother wavelets for WT and IMFs for HHT) are superposed linearly. As will be shown by the following results, even HHT cannot cope properly and quantitatively with the nonlinear phenomena involved in the original raw signals although Huang et al. (1998) initially developed HHT for the analysis of both non-stationary and nonlinear time series analysis.

The main objective of this paper is a comparative study of the results obtained by analysing the same wave signals obtained from laboratory experiments on the wave transformation over a submerged structure with a simple geometry using FFT, WT and HHT. Based on a closer examination of the effect of the applied analysis method on the results, the advantages of the joint time-frequency analysis methods (WT and HHT) as compared to the much simpler standard frequency analysis (FFT) will be identified in more detail. Moreover, the results will help to reveal more clearly, that non-linear effects cannot be currently resolved and that non-linear analysis methods will be required for this purpose.

COMPARATIVE ANALYSIS USING FFT, WT AND HHT

When waves interact with an artificial reef, the transmitted wave heights generally decrease and the transmitted wave periods change due to local and global energy losses as well as to energy shift within the wave spectrum, including wave breaking, nonlinear interactions and vortex generation (Bleck and Oumeraci, 2002). Due to these effects the generation of additional wave components is expected. Small scale hydraulic model tests (see Fig. 1) were conducted at Leichtweiss-Institute for Hydraulic Engineering (LWI), Technical University Braunschweig (Oumeraci and Bleck 2000). The aim of these tests was to provide a complete physical description the hydraulic performance of narrow and wider artificial reefs (Oumeraci and Bleck 2000, Bleck and Oumeraci 2002). The comparative wave analysis is carried out below for the following structure and wave parameters: reef width l_r=1.00 m, reef height h_r=0.50 m, water depth in front and behind the reef d=0.70 m, water depth over the reef d_r=0.20 m (submergence depth), incident wave height H_S=0.12 m, wave period T_p=1.5 s, sampling rate f=40 Hz, wave gauges array 02 for the incident wave and wave gauges array 16 for the transmitted waves behind the structure (Fig. 1). For further details on the experimental set-up and the testing conditions refer to Oumeraci and Bleck (2000) and to Bleck and Oumeraci (2002).

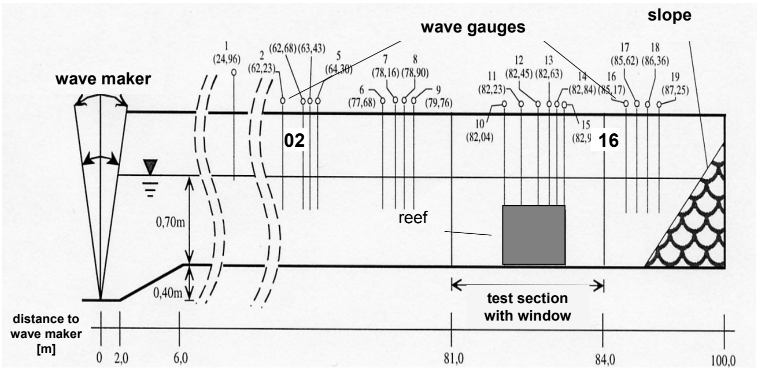

Fig. 1. Experimental set-up for artificial reef experiments at LWI (Oumeraci and Bleck 2000)

Fast Fourier Transform (FFT)

As mentioned in the introduction, Fourier analysis is the simplest and most widely used method of analysis of time series. Its simplicity arise from the fact that sine and cosine waves are selected prior to analysis as basic components which are then superposed linearly.

Because FFT is strictly valid for stationary signals, the basic issue in applying Fourier analysis to non-stationary time series is the trade-off between a window narrow enough to fulfil approximately the stationary condition and a window which should be wide enough to avoid obtaining a meaningless spectrum which has no relations with the spectrum of the original time series. Although Short Time Fourier Transform (STFT) allows to a certain extent a time-frequency description, the common FFT is limited to the frequency representation only.

Regular Wave Trains

Due to shallow water and wave flume effects, the incident regular wave differs from a pure sine wave. In the incident Fourier spectrum (Fig. 2a) the generated regular peak frequency $f_p = 0.66$ Hz is identified clearly. Due to the mentioned deviation from sine shape, the FFT considers the incident regular wave as a superposition of two different sine waves with different frequencies, f_p and $f = 2 \cdot f_p = 1.33$ Hz.

As a wave passes over a submerged structure it is expected that the wave height will decrease and that additional harmonic wave components will be generated behind the structure. Both effects are shown in the transformed FFT wave spectrum (Fig. 2b). The incident peak frequency amplitude decreased from 0.046 m·s in Fig. 2a to 0.03 m·s in Fig. 2b. While passing the reef, the incident wave has lost energy due to local and global dissipation effects. As a result, the wave height decreased. Behind the reef, at the higher harmonic frequencies $f = 2 \cdot f_p$ and $f = 3 \cdot f_p$; $4 \cdot f_p$ $5 \cdot f_p$ and $6 \cdot f_p$, the wave energy increased. In addition to these clearly identified higher frequencies numerous other non-harmonic frequencies with very small amplitudes are shown over the entire spectrum. These small amplitudes in the FT spectrum may be interpreted as mathematical artefacts which are required to reconstruct the non-sinusoidal wave components involved in the transmitted time series.

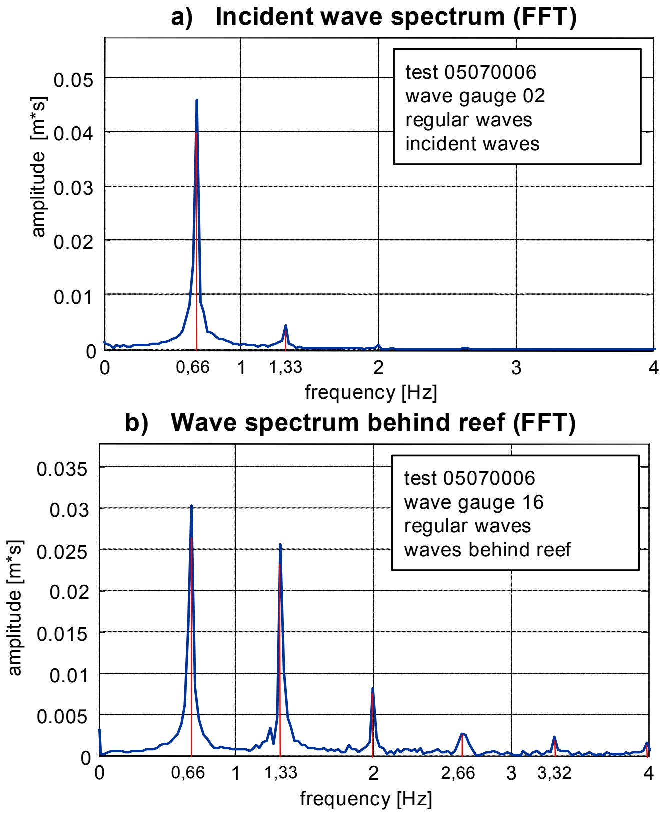

Fig. 2. Incident and transmitted regular waves' spectra over the reef using FFT

Irregular Wave Trains

The Fourier analysis of wave spectra passing the reef (Figs. 3a and 3b) leads to similar results. The incident spectrum shows a wide signal around the frequency f = 0.66 Hz. In contrast to Fig. 2a no secondary wave component with f = 1.33 Hz is identified. Since the in situ recorded wave spectrum is a superposition of numerous different wave components, FFT requires numerous wave components between f = 0.5 Hz and f = 1.5 Hz to reconstruct this wave train. Similar to the regular FFT spectrum, a slight loss of energy around the peak frequency f_p is shown in the transmitted spectrum (Fig. 3b): the amplitude decreases from 0.01 m·s to 0.007 m·s. Broad significant peaks are depicted at $f = 2 \cdot f_p = 1.36$ Hz and $f = 3 \cdot f_p = 2{,}0$ Hz in the transmitted spectra. Harmonic components higher than $f = 3 \cdot f_p$ are not observed.

Discussion of FFT-results

Overall, the FFT spectra in Figs. 3a and 3b has successfully shown that the wave energy is indeed transferred from low to high frequency wave components.Since the FFT processes the time series as a linear superposition of sine and cosine waves, neither nonlinear nor non-stationary signals are explicitly considered. Non-stationary frequencies contained in the time series, especially in the wave spectrum, are represented by stationary sine waves. Consequently, in the FFT algorithm artificial wave components have to be added which do not represent physical reality in the frequency spectrum.

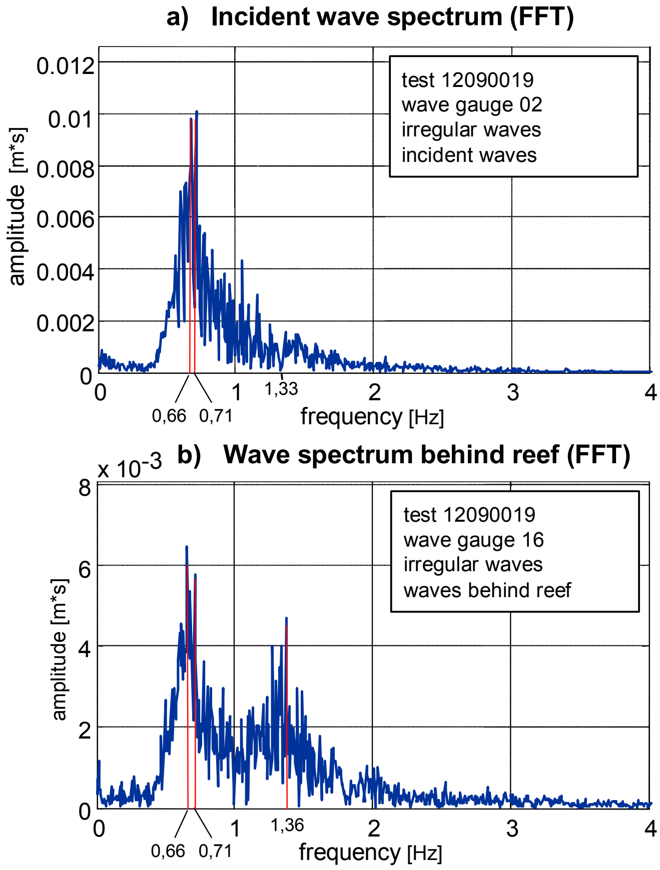

Fig. 3. Incident and transmitted irregular wave spectra over the reef using FFT

Wavelet Analysis (WT)

Wavelet represents the most widely used joint time-frequency analysis method (Massel 2001). Unlike STFT which uses a single analysis window, WT uses short windows at high frequencies and long windows at low frequencies, thus leading to a multi-resolution analysis; i.e. both frequency and time resolution vary in the time-frequency domain without violating the so called "uncertainty principle". While STFT uses sine and cosine signals multiplied by a sliding window as basis components, WT applies the window itself which is translated and dilated arbitrarily (mother wavelet) as a basis component. Despite a better resolution of non-stationary signals, the basic limitation which is also inherent to Fourier analysis remain as the basic components, which are fixed prior to analysis, are superposed linearly.

Regular Wave Trains

Prior to the WT analysis, the Morlet wavelet is chosen as basis wavelet. Just like the incident FFT spectrum, the WT spectrum in Fig. 4a shows a peak frequency of $f_p = 0.66$ Hz and a weak secondary wave component with $f = 2 \cdot f_p = 1.33$ Hz. The transmitted spectrum in Fig. 4b also confirms the FFT results. The incident peak frequency amplitude decreases about 30% while the amplitude of the secondary component with $f = 2 \cdot f_p$ increases. The energy transfer within the spectrum as described by FFT is confirmed. The generation of harmonics higher than $f = 2 \cdot f_p$ as contained in the FFT spectrum can, however, not be identified in the result of WT-analysis.

Irregular Wave Trains

For irregular wave trains (Figs. 5a and 5b) the incident peak frequency f_p is well-identified. The broad peak confirms the existence of several wave components between f = 0.5 Hz and f = 1,5 Hz. Behind the reef, the FFT results are confirmed: At the peak frequency wave energy decreases while it increases at frequency $f = 2 \cdot f_p = 1.30$ Hz.

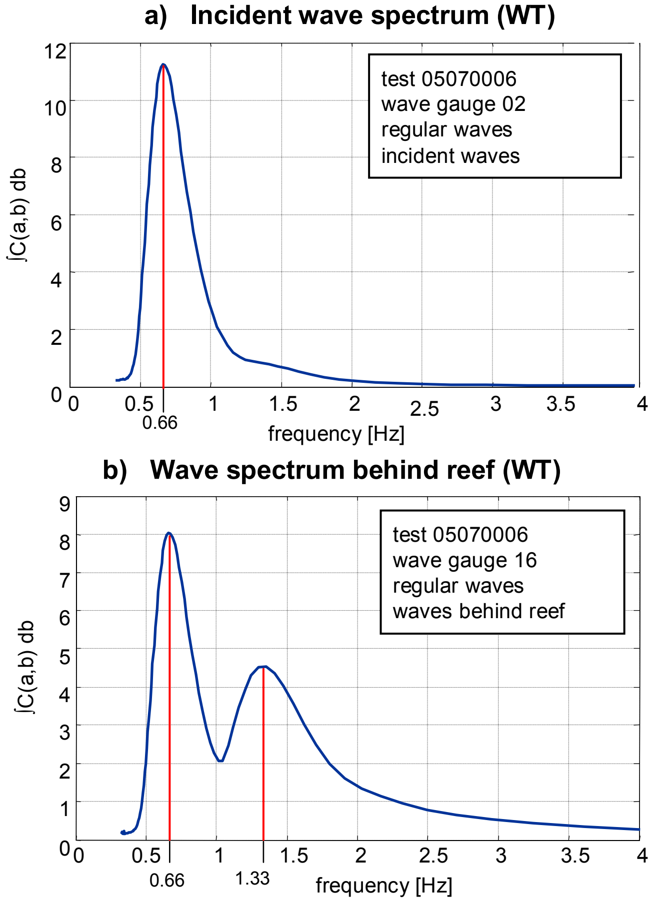

Fig. 4. Incident and transmitted regular wave spectra over the reef using Wavelet (WT)

Discussion of WT-results

The Morlet wavelet, which is usually used for ocean wave analysis, is generated by linearly superposed finite sine waves. Non-stationary effects within the range of a wavelet are not considered. Non-stationary frequency variations can be detected and assigned to their time of occurrence. Therefore, the spectrum shows a wide peak instead of a narrow signal. The shape of the basis wavelet has to be selected prior to the analysis. This choice significantly affects the result and depends on the different wave trains considered as well as on the experience of the analyst. In addition to the frequency spectrum, WT provides a time-frequency diagram that associates every detected frequency with its time of occurrence. The most serious limitation of FFT, its application to stationary signals, is removed when applying WT.

Hilbert-Huang-Transform (HHT)

HHT represents the most recent joint time-frequency analysis method. Unlike WT which lacks adaptivity because the basic components (mother wavelets) are fixed prior to analysis, HHT uses Intrinsic Mode Functions (IMF) as basic components. The latter are directly obtained from the original raw signal by applying the so-called Empirical Mode Decomposition (EMD). The key feature of HHT is the sifting process to generate IMFs. As a result of this process, the complicated time series are reduced into frequency- and amplitude-modulated form, enabling the instantaneous frequency to be defined. This indeed makes HHT the first local and adaptive tool among the yet existing time-frequency analysis methods. However, like for FFT and WT, the inherent limitation remains its inability to cope properly and quantitatively with nonlinear phenomena involved in the time series, although HHT has initially been developed for the analysis of nonlinear signals (Huang et al. 1998). This is not surprising as the IMFs are superposed linearly without any account of nonlinear interactions.

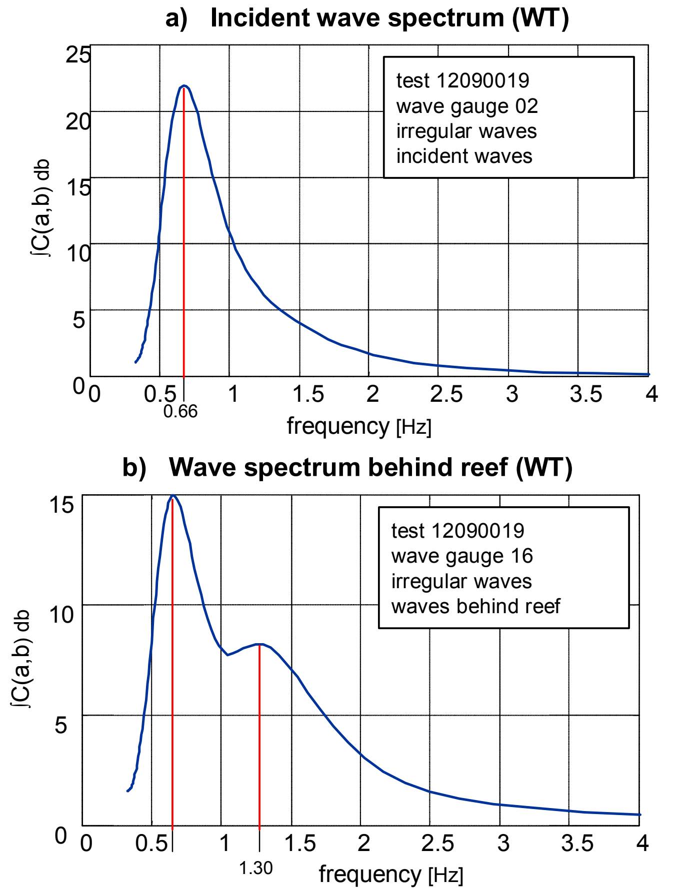

Fig. 5. Incident and transmitted irregular wave spectra over the reef using Wavelet (WT)

Regular Wave Trains

In the HHT analysis, which includes the Empirical Mode Decomposition (EMD) to pre-process the raw data, the measured wave train is considered as a linear superposition of Intrinsic Mode Functions (IMFs). Fig. 6a shows the marginal Hilbert spectrum of the incident wave train with a peak at $f_p = 0.65$ Hz and a secondary peak at $f = 0.73$ Hz. The incident wave peak frequency is reliably identified, and the signal obtained by the HHT analysis is narrower than in the WT spectrum. The $2 \cdot f_p$-component confirmed by FFT and WT is not confirmed by HHT.

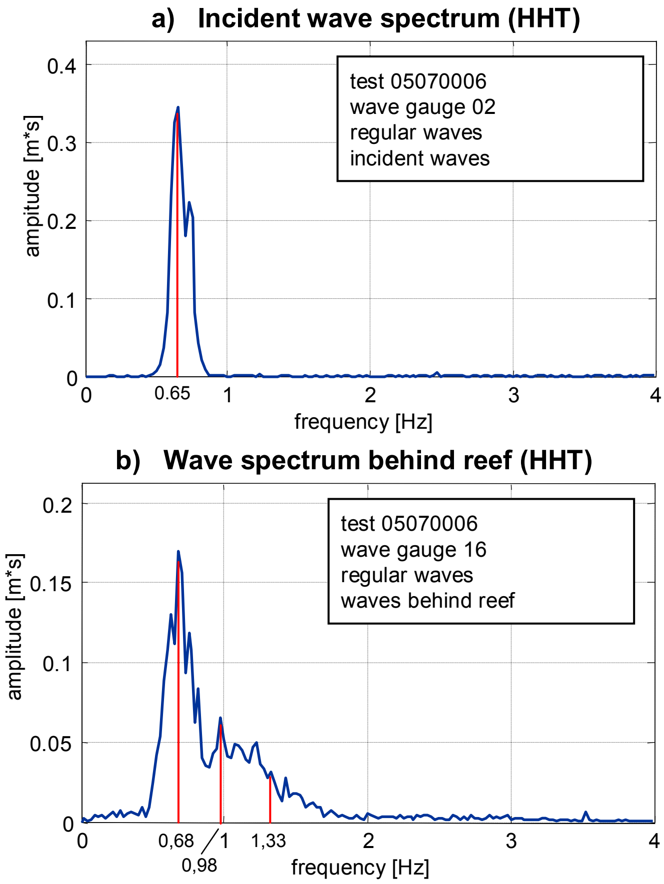

Fig. 6. Incident and transmitted regular wave spectra over the reef using Hilbert-Huang-Transform (HHT)

Behind the reef (Fig. 6b) three frequencies are identified ($f_p = 0.68$ Hz, $f = 0.98$ Hz and $f = 2 \cdot f_p = 1.33$ Hz). The component with $f = 0.98$ Hz possibly represents an artefact due to non-optimised EMD. A serious problem of the WT and FFT has been overcome by HHT as the basic components (IMFs) are derived directly from the actual wave train. The identified basis components may be non-stationary, but they are superposed linearly to retrieve the original wave train.

Irregular Wave Trains

Similar to FFT and WT, the HHT wave spectra analysis determines an incident peak frequency of $f_p = 0.67$ Hz. Behind the reef, the peak frequency is $f_p = 0.63$ Hz and a higher harmonic with $f = 2 \cdot f_p = 1.25$ Hz is detected. The frequency peak at $f = 1.08$ Hz is possibly an artefact due to non-optimised EMD. A further discussion of the results is also given by Schlurmann et al. (2002).

Discussion of HHT-results as composed to FFT and WT results

The three applied frequency analysis methods have confirmed the energy transfer and the generation of higher frequency wave components behind the reef. In every transmitted spectrum the incident wave peak frequency is the most powerful component and new frequencies with $f = 2 \cdot f_p$ are identified. Unlike FFT, WT and HHT do not show components with $f > 2 \cdot f_p$, so that these high frequency waves might be regarded as artefacts. Unlike FFT and WT, HHT identifies a wave component with $f \approx 1.0$ Hz. At this stage, it is impossible to decide, whether this component is an artefact due to non-optimised EMD algorithm or whether it is identified correctly.

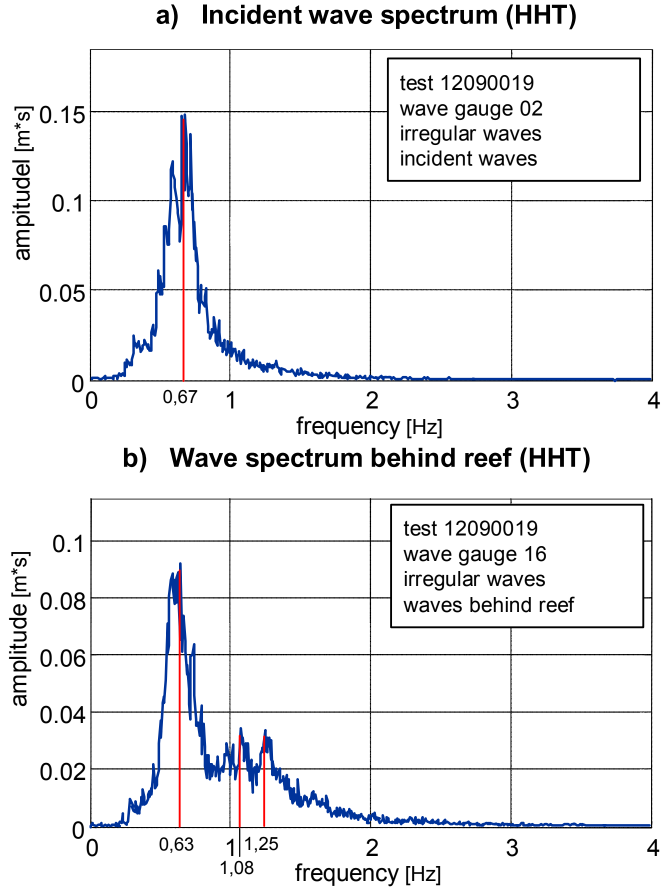

Fig. 7. Incident and transmitted irregular wave spectra over the reef using Hilbert-Huang-Transform (HHT)

CONCLUDING REMARKS AND PERSPECTIVES

The comparative analysis of FFT, WT and HHT as applied to the simple example of regular and irregular wave trains transmitted over an impermeable reef with a simple geometry has shown that:

- The peak frequency of the incident waves is reliably identified by all three methods.
- The energy transfer from the incident peak frequency to higher frequencies within the transmitted wave spectrum is well confirmed by all three methods.
- Unlike WT and HHT, Fast Fourier Transform depicts more higher frequency components which might be considered as mathematical artefacts as FFT is unable to resolve both non-stationary and nonlinear effects.
- Wavelet Transform (WT) and Hilbert-Huang-Transform (HHT), which are able to account for non-stationary signals shows almost similar results although WT lacks adaptivity as the mother wavelets (basic components) are fixed prior to analysis. Because HHT uses Intrinsic Mode Functions (IMF) as basic components directly obtained from the original raw signals, a much better performance of HHT would have been expected.
- Although WT and HHT proved more performant than FFT, the key limitation which is inherent to all three methods has been revealed by the

results of the example application: the linearity of the applied methods themselves. The basic components (sine and cosine waves in FFT, wavelets in WT, IMFs in HHT) are superposed linearly and the nonlinear interaction between the basic components are not considered. To better understand the nonlinear processes which are particularly associated with shallow water waves and wave structure interaction a nonlinear analysis method using nonlinear waves as basis components and accounting for their nonlinear interaction is urgently needed.

A similar study using wave signals obtained from laboratory experiments on wave transformation over a sloping structure (Oumeraci and Muttray 2001) could not be included in this paper for lack of space, but the results entirely support the conclusions drawn from the artificial reef experiments analysed in this paper (Bruehl 2003).

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