Ocean Waves Measurement and Analysis, Fifth International Symposium WAVES 2005, 3rd-7th, July, 2005. Madrid, Spain
Paper number: 133

ON GENERATION OF SINGLE STEEP WAVES IN TANKS

Lev Shemer[1], Konstantin Goulitski[2], Eliezer Kit[3], Joachim Grüne[4] and Reinold Schmidt-Koppenhagen[5]

Abstract: Very steep waves constitute an essentially nonlinear and complicated phenomenon. Hence, inter-related experimental and theoretical efforts are required to gain a better understanding of their generation and propagation mechanisms. To this end, a nonlinear focusing process in which a single unidirectional steep wave emerges from an initially wide amplitude- and frequency-modulated wave group at a predicted position in the laboratory wave tank is studied both theoretically and experimentally. The spatial version of the Zakharov equation was applied in the numerical simulations. Experiments were carried out in the 330 m long Large Wave Channel in Hannover. Quantitative comparison between the experimental and the corresponding numerical results is carried out. Good agreement is obtained between experiments and calculations. The effect of dissipation and bound waves is discussed.

INTRODUCTION

To model tsunami effect in laboratory conditions, an ability to excite a single steep wave is required. Generation of very steep waves in wave tanks enables experimental study of the tsunami wave damage potential and is thus of great importance. Excitation of single steep wave at a prescribed location in a laboratory wave tank of constant depth is also often required for model testing in coastal and ocean engineering. It is well known that such waves can be generated by focusing a large number of waves at a given location and instant. Dispersive properties of deep or intermediate-depth surface gravity waves can be utilized for this purpose. Since longer gravity waves propagate faster, a wave group generated at the wave maker in which wave length increases from front to tail may be designed to focus the wave energy at a desired location. Such a

---

1 Professor, Department of Fluid Mechanics, Faculty of Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel shemer@eng.tau.ac.il
2 Department of Fluid Mechanics, Faculty of Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel konst16@eng.tau.ac.il
3 Professor, Department of Fluid Mechanics, Faculty of Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel kit@eng.tau.ac.il
4 Coastal Research Centre FZK, Merkurstrasse 11, D-30419 Hannover, Germany gruene@fzk.uni-hannover.de
5 Coastal Research Centre FZK, Merkurstrasse 11, D-30419 Hannover, Germany sk@fzk.uni-hannover.de

wave sequence can be seen as a group that is modulated both in amplitude and in frequency. One-dimensional theory describing spatial and temporal focusing of various harmonics of dispersive gravity waves based on the linear Schrödinger equation was suggested by Pelinovsky & Kharif (2000). These authors suggested such a focusing as a possible mechanism responsible for generation of extremely steep singular waves. However, the experiments of Brown & Jensen (2001) demonstrated that nonlinear effects are essential in the evolution of those waves. An extensive review of field observations of those waves, as well as relevant theoretical, numerical and experimental studies was recently presented by Kharif and Pelinovsky (2003).

The essential nonlinear behavior of wave groups with high maximum wave steepness has been demonstrated in a number of studies. Attempts were made to describe the propagation of deep or intermediate depth gravity water-wave groups with a relatively narrow initial spectrum by a cubic Schrödinger equation (CSE), Shemer et al. (1998). It was demonstrated in this study that while CSE is adequate for description of the global properties of the group envelope evolution, it is incapable to capture more subtle features such as the emerging front-tail asymmetry observed in experiments. For the weakly-dispersive wave groups in shallow water, application of the Korteweg – deVries equation provided results that were in very good agreement with the experiments (Kit et al. 2000). In the case of stronger dispersion in deeper water, more advanced models than the CSE are required. This stems from the fact that due to nonlinear interaction, considerable widening of the initially narrow spectrum is observed. The modified Schrödinger equation due to Dysthe (1979) is a higher (4^{th}) order extension of the CSE. Application of this model indeed provided good agreement with experiments (Shemer et al. 2002). An alternative theoretical model that is free of band-width constraints is the Zakharov (1968) equation. Unidirectional spatial version of this equation was derived in Shemer et al. (2001) and applied successfully do describe the evolution of nonlinear wave groups in the tank. Kit & Shemer (2002) showed the relation between the spatial versions of the Dysthe and the Zakharov equations.

An attempt to check the limits of applicability of the Dysthe equation to describe evolution of wave groups with wider spectrum has been carried out in Shemer et al. (2002). Numerical solutions of the wave group evolution problem were carried out using both the Dysthe model and the Zakharov equation. The obtained results demonstrated that while the Dysthe model performed in a satisfactory fashion for not too wide spectra, it failed when initially very wide spectra were used.

Extremely steep (freak) wave can be seen as a wave group with a very narrow envelope and correspondingly wide spectrum. Kharif et al. (2001) applied the cubic Schrödinger equation (CSE) model to simulate nonlinear freak wave generation by the focusing mechanism. For the reasons presented above, numerical simulation of this problem requires, however, application of a model without strong bandwidth limitations. In the current study we perform an experimental study of propagation of steep wave groups with wide spectrum in a large wave tank, accompanied by numerical simulations based on the spatial version of the Zakharov equation.

THEORY

The purpose of the present study is to obtain at a prescribed distance from the wavemaker, $x = x_0$, steep unidirectional wave group with a narrow, Gaussian-shaped envelope with the surface elevation variation in time, $\zeta(t)$, given by

$$\zeta(t) = \zeta_o \, exp - (t / mT_0)^2 \, cos(\omega_o t) \tag{1}$$

where $\omega_0=2\pi/T_0$ is the carrier wave frequency, ζ_0 is the maximum wave amplitude in the group, and the parameter m defines the width of the group. The small parameter representing the magnitude of nonlinearity ε is the maximum wave steepness $\varepsilon = \zeta_0 k_0$. The wave number is related to the frequency ω by the finite depth dispersion relation

$$\omega^2 = kg \tanh(kh), \tag{2}$$

g being the acceleration due to gravity. The parameter m determines the width of the group; higher values of m correspond to wider groups and consequently narrower spectra. The spectrum of the surface elevation given by (1) is also Gaussian.

To produce single steep wave at a desired location along the tank, we apply the “time-reversal” idea suggested by Pelinovsky and Kharif (2000). The wave field at earlier locations, $x < x_0$ is obtained from the computed complex surface elevation frequency spectrum at this location. To this end, the unidirectional discretized spatial version of the Zakharov equation is used:

$$i \cdot c_g \frac{\partial B_j(x)}{\partial x} = \sum_{\omega_j+\omega_l=\omega_m+\omega_n} V(\omega_j,\omega_l,\omega_m,\omega_n) B_l^* B_m B_n \, e^{-i(k_j+k_l-k_m-k_n)x} \tag{3}$$

where c_g is the group velocity and * denotes complex conjugate. This equation describes the slow evolution along the tank of every free spectral component $B_j = B(\omega_j)$ of the surface elevation spectrum in inviscid fluid of constant (infinite or finite) depth and accounts for Class I, or quartet, nonlinear interactions among various components. The procedure for computation of the interaction coefficients V in (3) is based on Krasitskii (1994) and was developed by Annenkov (2002).

The dependent variables $B(\omega_j, x)$ representing the free components in the wave field, are related to the generalized complex ‘amplitudes’ $b(\omega_j, x)$. These complex amplitudes are composed of the Fourier transforms of the surface elevation $\hat{\zeta}(\omega_j, x)$ and of the velocity potential at the free surface $\hat{\phi}^s(\omega_j, x)$:

$$b(\omega,x) = \left(\frac{g}{2\omega}\right)^{1/2} \hat{\zeta}(\omega,x) + i\left(\frac{\omega}{2g}\right)^{1/2} \hat{\phi}^s(\omega,x) \tag{4}$$

The amplitude b represents a sum of the free and the bound waves given by

$$b(\omega_j, x) = [\varepsilon B(\omega_j, x_2) + \varepsilon^2 B'(\omega_j, x, x_2) + \varepsilon^3 B''(\omega_j, x, x_2)\ldots]\exp(ikx) \tag{5}$$

The bound higher order components B' and B'' can be computed at each location once the free wave solution $B_j(x)$ is known. The corresponding formulae, as well as the kernels necessary for their computations are given in Krasitskii (1994). The scaled coordinate $x_2 = \varepsilon^2 x$. Inversion of (4) allows computing the Fourier components of the surface elevation $\hat{\zeta}(\omega,x)$. Inverse Fourier transform then yields the temporal variation for the surface elevation $\zeta(x,t)$.

The spectrum corresponding to (1) is integrated using (3) from the planned focusing location x_0 backwards up to the wavemaker x = 0. The waveforms derived from the computed spectra serve as the wavemaker driving signals, with corrections that account for the actual wavemaker response. The study is carried out for various maximum wave amplitudes.

EXPERIMENTAL FACILITY AND PROCEDURE

The wave tank in Hannover has length of 330 m, width of 5 m and depth of 7 m. Water depth in the present experiments was set to be 5 m. At the end of the wave tank there is a sand beach starting at the distance of 270 m with slope of 30°. The piston-type wavemaker is driven by a computer-generated signal and is equipped with the reflected wave energy absorption system. The focusing location in all experiments was set at 120 m from the wavemaker. The instantaneous water height in the wave tank is measured using 25 wave gauges of resistance type, which are placed along the wave tank wall; higher concentration of the wave gauges is in the region of expected focusing of the wave group. For technical reasons, those gauges can only be placed at distances starting from about 50 m from the wavemaker. Besides, there are two additional gauges, one at the wavemaker at the distance of 0.05 m from the wavemaker plane, and second at the distance of 3.45 m. The calibration of the gauges was carried out during filling the wave tank using 8 depths that resulted in an effective surface elevation range of ±0.8m. The accuracy of water height measurements is ±5 mm; the calibration curve is linear, with coefficients determined separately for each gauge. During experiment, signals from wave gauges are sampled at the frequency of 40 Hz for each probe for total duration of 350 s.

The carrier wave period adopted in (1) is T_0 = 2.8 s, corresponding to the wavenumber k_0= 0.52 m^{-1}, so that k_0h = 2.59 and thus deep-water dispersion relation is approximately satisfied. The lower harmonics of the spectrum, contrary to the carrier wave, do not satisfy the deep water condition anymore. Therefore, in all expressions for the interaction coefficients finite depth versions were used in this study. The maximum driving amplitudes are selected so that at the focusing location, the resulting carrier wave has the maximum wave amplitudes ζ_0 corresponding to the steepness $\varepsilon = k_0\zeta_0$ ranging from ε = 0.1 to ε = 0.3.

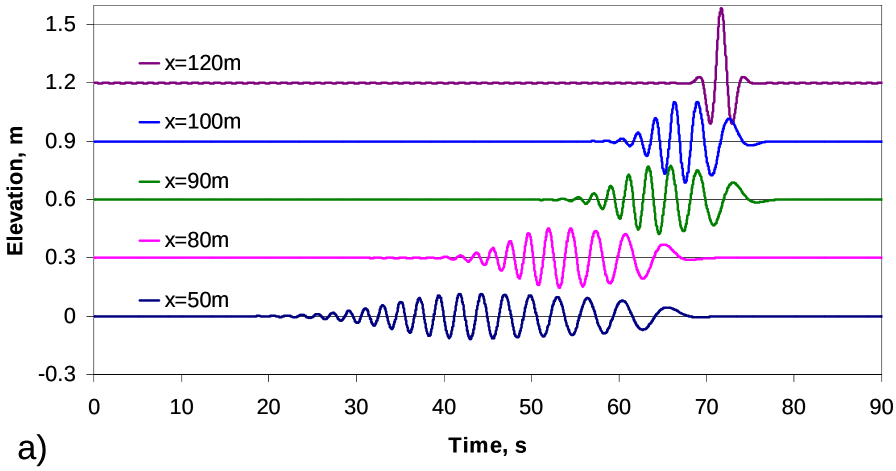
The Gaussian energy spectrum of (1) has a shape with the relative width at the energy level of ½ of the spectrum maximum that depends on the value of the parameter m in (1) and is given by

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{m\pi}\sqrt{\tfrac{1}{2}\ln 2} \tag{6}$$

The value of the group width parameter in the experiments was selected to be m=0.6, so that the relative spectrum width $\Delta\omega/\omega_0 = 0.312$, which is beyond the domain of applicability of the narrow spectrum assumption of the cubic Schrödinger and Dysthe models.

For each set of selected variable parameters (maximum steepness ε and the distance of the focusing location from the wavemaker x_0) the solution of the system of N ordinary differential equations (3), N being the total number of wave harmonics considered, was obtained for distances from the wavemaker up to x= 120 m. The number of free wave harmonics considered in this study is $N = 100$. The wavemaker driving signal was adjusted to get as good as possible agreement between the calculated and the measured wave field at $x = 50$ m. This value of x was chosen since it is the closest to the wavemaker measuring location where the evanescent modes do not contaminate the wave field.

RESULTS



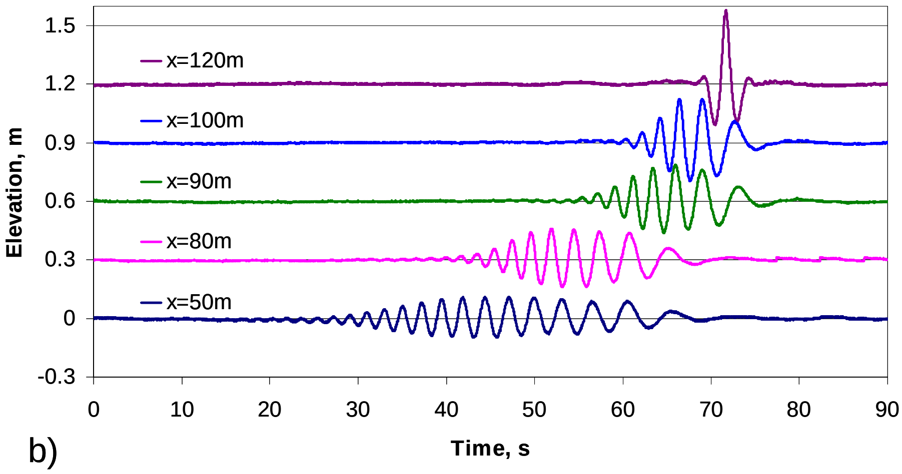


Fig. 1. Temporal variation of the surface elevation along the tank for x_0=120 m and $\varsigma_0 k_0$=0.2: a) computed; b) measured.

A representative selection of the accumulated in this study cases is discussed in this Section. In the first case considered the maximum wave steepness at the focusing location $\zeta_0 k_0$ =0.2. The computed and the measured temporal variation of the surface elevation at different locations along the tank are presented in Figs. 1a and 1b, respectively. From the computation results in Fig. 1a one can see that the selected value of the group-width parameter m in (1) indeed yields a narrow wave group with a single steep wave at this location. Closer to the wavemaker the group becomes notably wider, and the maximum wave amplitudes decrease accordingly. The amplitude and the frequency modulation within the group are clearly seen. The experimental results presented in Fig. 1b demonstrate a very good agreement with the computations. Residual noise is observed behind the group, and the wave shape at the focusing location is not exactly symmetric.

The computed and the measured spectra for the experimental parameters of Fig. 1 are presented in Fig. 2 at various locations along the tank. The variation of the spectral shape along the tank is evident and indicates that wave evolution is essentially nonlinear even at this relatively low amplitude of forcing. The agreement between experiments and computations is quite satisfactory and both the numerical simulations and the measurements exhibit similar features. The spectral shapes shown in Fig. 2b indicate that the spectrum becomes wider with the distance from the wavemaker and at the prescribed distance of $x_0 = 120$ m approaches the Gaussian shape of the numerical simulations. The peak frequency at $x = 50$ m is shifted to the right relative to the carrier frequency $f_0=1/T_0$. The low frequency part of the spectrum remains unaffected during the evolution process.

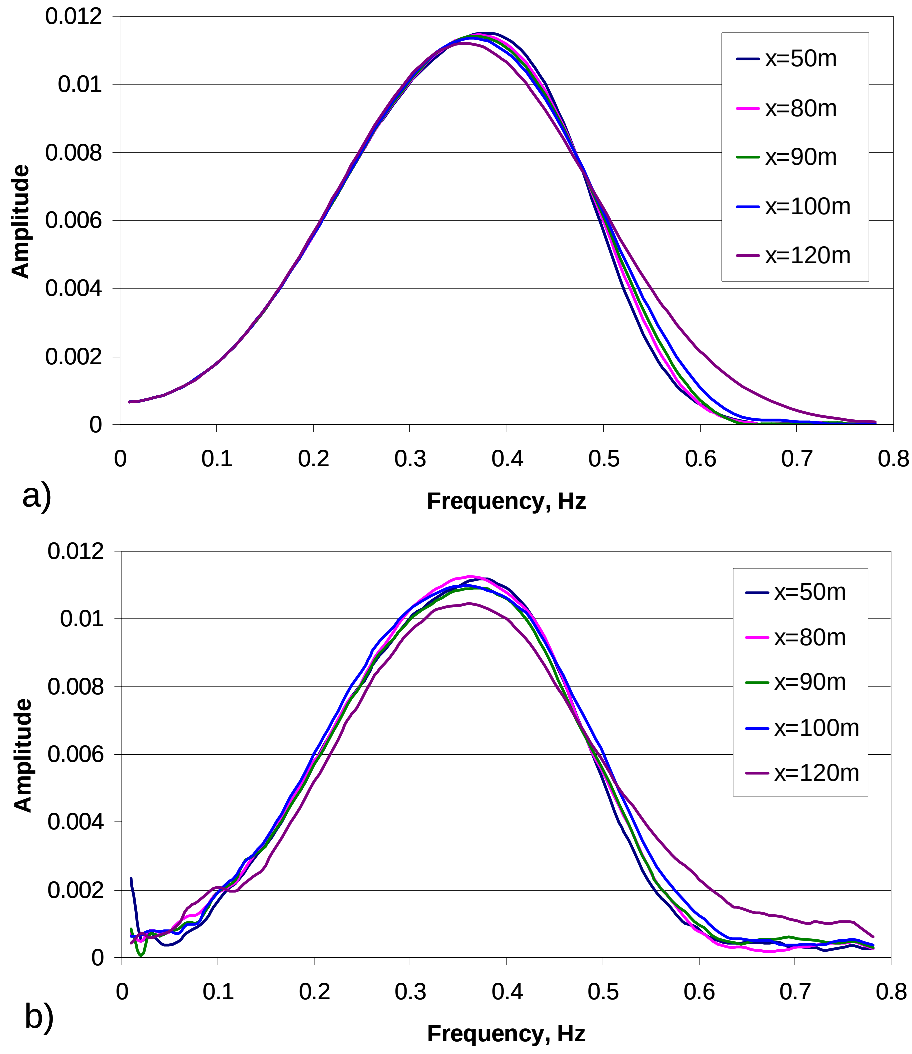
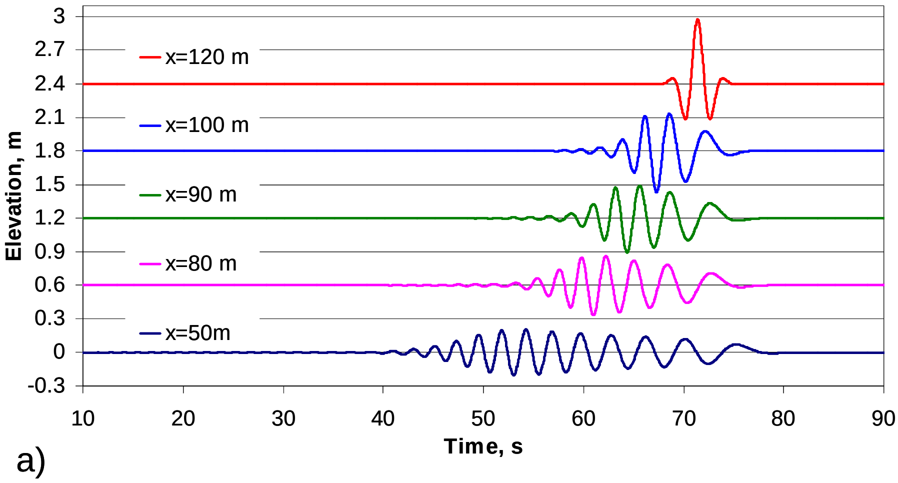
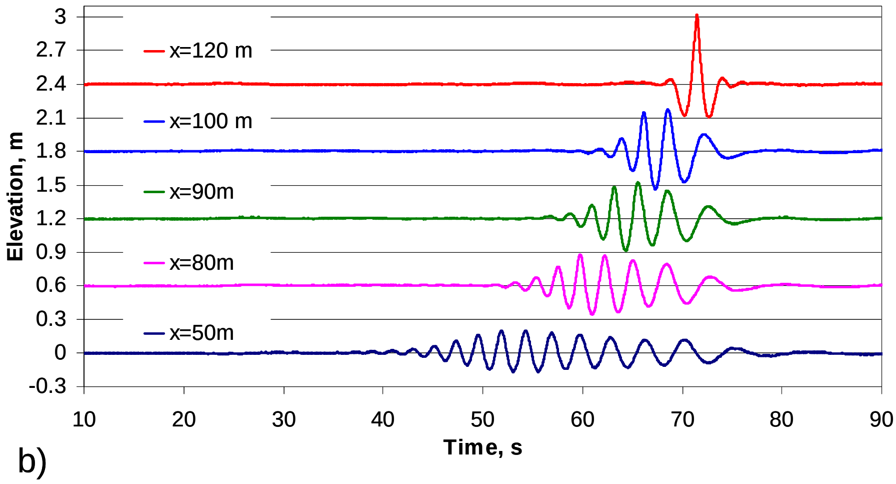


Fig. 2. Frequency spectra of the surface elevation variation with time at a number of locations along the tank for x_0=120 m and $\varsigma_0 k_0$=0.2: a) computed; b) measured.

When the maximum wave steepness is increased to $\zeta_0 k_0 = 0.3$, the agreement between the computed (Fig. 3a) and the measured (Fig. 3b) surface elevation along the tank remains quite good. The distortion of the envelope shape at x = 50 m is visible in computations as well as in experiment. Note that the peak values within the group appear to be somewhat different in those figures. It should be stressed that the computed surface elevation is obtained taking into account the free modes only, while in the experiments the effect of the bound waves can be of significance.



a)



b)

Fig. 3. Temporal variation for the surface elevation along the tank for x_0=120 m and $\varsigma_0 k_0$=0.3: a) computed; b) measured.

Additional reason for certain disagreement between the measured results and the computations can be attributed to the difficulties to reproduce accurately in the experiments the computed spectrum at the vicinity of the wavemaker. The results on the wave amplitude spectra for the conditions of Fig. 3 are given in Fig. 4. Comparison of the variation of the computed spectral shapes at moderate maximum wave steepness in Fig. 2a and in Fig. 4a for a very steep wave group clearly indicates that the evolution process in the latter case is strongly affected by nonlinearity. The agreement between the measured and the computed spectra is now less impressive. This can be partially attributed to difficulties in accurate reproduction in the experiment of the quite a complicated spectral shape obtained at x = 50 m (Fig. 4a). While the lower frequency parts of the spectra are quite similar in computations and in the experiments, the higher-frequency spectral amplitudes in the experiment decay much faster than in the computations. This can possibly stem from the energy dissipation of the shorter waves due to both viscous effects and breaking.

It is customary in ocean engineering to determine wave height, H, as the difference between the consecutive minimum and maximum surface elevation. We adopt this definition here to study the wave height evolution along the tank. The evolution of the maximum wave height, H_{max}, within the group along the tank for ε = 0.2 is shown in Fig. 5a for the focusing location of x_0 = 5 m. Although there is a consistent shift between the measured and the computed values due to the difficulties in accurate generation at the wavemaker of the required initial waveform, the rate of increase of the maximum wave height during the focusing process and the following decrease in the maximum wave height during defocusing for $x > x_0$ are practically identical.

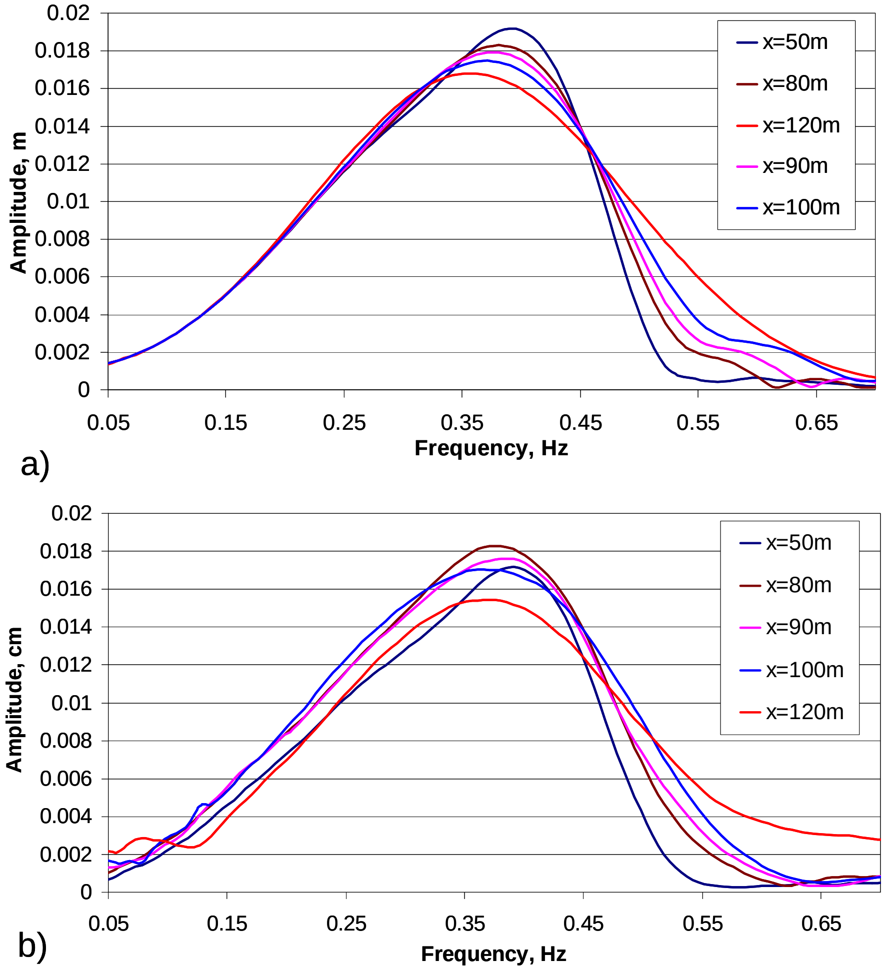


Fig. 4. Frequency spectra of the surface elevation variation with time at a number of locations along the tank for x_0=120 m and $\varsigma_0 k_0$=0.3: a) computed; b) measured.

At an essentially higher amplitude (ε = 0.4), the disagreement between the computational and the experimental results shown in Fig. 5b for x_0 = 120 m is clearly visible. In the theoretical computations the growing and the decaying branches remain symmetric relative to the focusing point x_0. In the experiments, the growth rate of the maximum wave steepness is somewhat lower than in simulations, suggesting stronger dissipation. The notable decay of the measured value of H_{max} following the focusing location x_0 is an indication that this very steep wave breaks.

CONCLUDING REMARKS

The ability to obtain focused steep waves at any desired location along the tank is demonstrated. It is shown that the focusing process is accompanied by a notable change of the spectral shape and is thus essentially nonlinear. The unidirectional spatial discrete version of the Zakharov equation is adequate to describe nonlinear evolution of wave groups with wide spectrum and moderate steepness. For very steep waves, the agreement between the model predictions and the experiments is only qualitative. This is attributed to a number of effects. The reproduction in the experiments of the desired waveform at the wavemaker becomes more complicated as the amplitude increases.

The effects related to the bound waves strongly depend of the wave steepness. These effects in principle can be accounted for in the framework of the Zakharov equation. The corrections due to bound waves were not introduced in the current study. The most important factor, however, that causes the discrepancy between the experiments and the computations, is the wave breaking. The breaking effects and the resulting energy dissipation were clearly documented in the present study. The Hamiltonian Zakharov model is incapable of describing those non-conservative effects.

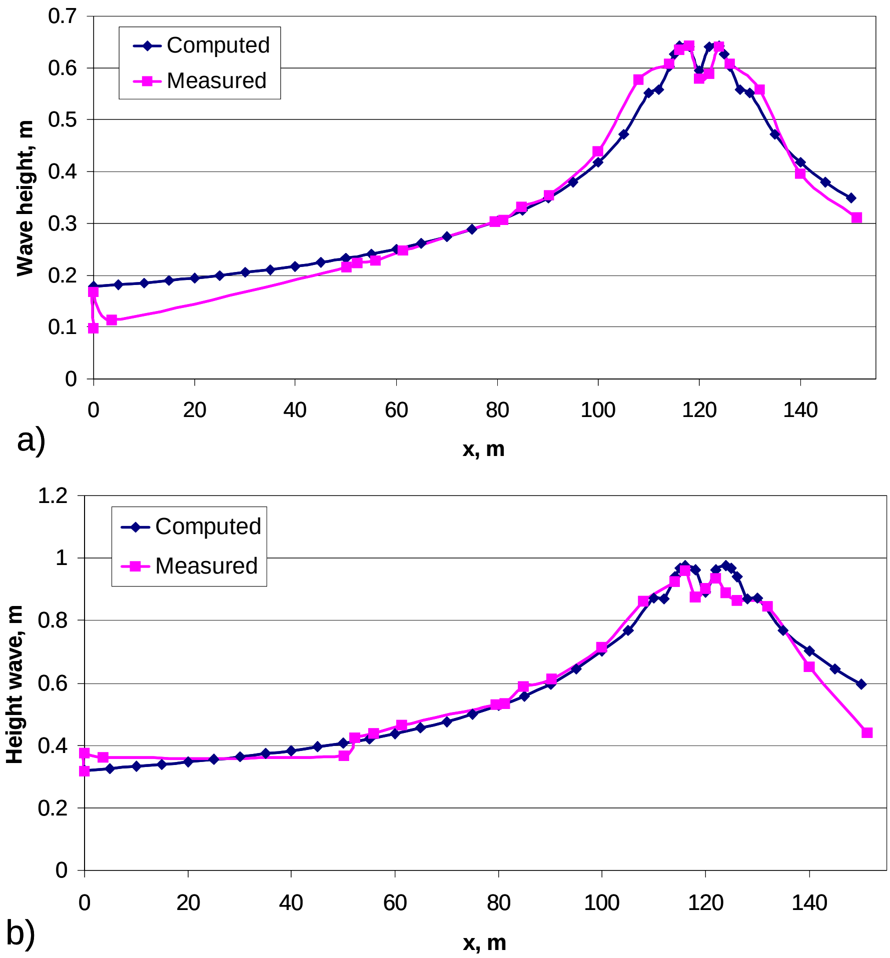


Fig. 5. Measured and computed variation along the tank of the maximum wave height within the group x_0=120m. a) $\varsigma_0 k_0$=0.2; b) $\varsigma_0 k_0$=0.3.

ACKNOWLEDGEMENT

The authors acknowledge the support provided by European Community under the Access to Research Infrastructures Action of the Human Potential Programme (contract HPRI-CT-2001-00157) that made possible experiments in the Large Wave Channel (GWK) of the Coastal Research Center (FZK) in Hannover.

REFERENCES

Brown, M.G. and Jensen, A. 2001. Experiments on focusing unidirectional water waves. *J. Geophys. Res.,* 106, 16,917-16,928

Dysthe, K.B. 1979. Note on a modification to the nonlinear Schrödinger equation for application to deep water waves. *Proc. Roy. Soc. London,* A369, 105-114.

Kit, E., Shemer, L., Pelinovsky, E., Talipova, T., Eitan, O. and Jiao, H.-Y. 2000. Nonlinear wave group evolution in shallow water. *J. Waterway, Port, Coastal & Ocean Eng.*,126, 221-228.

Kit, E. and Shemer, L. 2002. Spatial versions of the Zakharov and Dysthe evolution equations for deep water gravity waves. *J. Fluid Mech.,* 450, 201-205.

Kharif, C. and Pelinovsky, E. 2003. Physical mechanisms of the rogue wave phenomenon. *Europ. J. Mech. B/Fluids,* 22, 603-634.

Kharif, C., Pelinovsky, E., Talipova, T. and Slunyaev, A. 2001. Focusing of nonlinear wave groups in deep water. *JETP Letters,* 73, 170-175

Krasitskii, V.P. 1994. On the reduced equations in the Hamiltonian theory of weakly nonlinear surface waves. *J. Fluid Mech.,* 272, 1-20.

Pelinovsky, E. and Kharif, C. 2000. Simplified model of the freak wave formation from the random wave field. *Proc. 15th Int. Workshop on Water Waves and Floating Bodies*, Caesaria, 142-145.

Shemer, L., Kit, E., Jiao, H.-Y., and Eitan, O. 1998. Experiments on nonlinear wave groups in intermediate water depth. *J. Waterway, Port, Coastal & Ocean Eng.,* 124, 320-327.

Shemer, L., Jiao, Haiying, Kit, E., and Agnon, Y. 2001. Evolution of a nonlinear wave field along a tank: experiments and numerical simulations based on the spatial Zakharov equation. *J. Fluid Mech.,* 427, 107-129.

Shemer, L., Kit, E., and Jiao, H.-Y. 2002. An experimental and numerical study of the spatial evolution of unidirectional nonlinear water-wave groups. *Phys. Fluids,* 14**,** 3380-3390.

Zakharov, V.E. 1968. Stability of periodic waves of finite amplitude on the surface of deep fluid. *J. Appl. Mech. Tech. Phys. (English transl.),* 2**,** 190-194.